Part A2 - **Merging and Stability of Electromagnetic Toroidal Structures**

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Abstract

This article investigates the merging, geometric growth, and stabilization of electromagnetic toroidal structures as a continuation of the model introduced in Part A1. The proposed mechanism considers the merging of geometrically similar, phase-compatible toroids into a unified higher-energy electromagnetic configuration.

Under the assumptions of constant geometric ratio and equilibrium energy density, the characteristic torus radii scale with the cube root of positive structural energy. Consequently, merging two identical toroids without radiation and without a change in their combined binding-energy contribution doubles their positive energy and volume while increasing both radii by approximately 26%, with the characteristic equilibrium electric-field amplitude remaining unchanged.

The analysis identifies the redirection of the Poynting vector as a possible connection between geometric deformation and electromagnetic restoring stresses. At constant energy, average energy density varies with the inverse third power of the geometric scale, while the characteristic local momentum-redirection force density varies with the inverse fourth power.

Electromagnetic momentum transport may produce transient oscillations following merging, while radiation can contribute to their damping and modify the equilibrium geometry. Within the proposed energy model, radiation may also reduce positive structural energy relative to approximately unchanged negative binding energy, thereby increasing relative binding dominance without increasing the absolute magnitude of the binding contribution.

The resulting framework connects torus merging, geometric relaxation, electromagnetic momentum transport, radiation, and energetic stabilization. It establishes quantitative scaling relationships and conditional stability criteria, while identifying the detailed spatial electromagnetic field configuration and the physical origin of negative binding energy as the principal requirements for demonstrating persistent structural stability. Larger electromagnetic transformations and possible supertorus formation are addressed in Part A3.

1. Introduction

The preceding article, Part A1, introduced the electromagnetic torus as a possible fundamental structural element of matter. It examined the spatial organization of electric and magnetic fields, their energy distribution, and the mechanisms that may maintain a stable toroidal configuration. Particular attention was given to the geometry of the structure, the internal flow of electromagnetic energy, and the balance between expansive and binding effects.

The present article, Part A2, extends this investigation from individual toroids to their mutual interaction, merging, and subsequent stabilization. The central hypothesis is that two electromagnetic toroids with sufficiently similar geometry, compatible field orientation, and appropriate phase alignment can merge into a single structure containing their combined energy. The resulting torus initially occupies a non-equilibrium state and must redistribute its electromagnetic energy to establish a new stable configuration.

The proposed model assumes that stable toroids tend toward a characteristic electric-field amplitude, while their geometric dimensions depend on the energy they contain. For geometrically similar toroids, the ratio between the minor radius a and major radius R remains constant. Consequently, the equilibrium dimensions increase with the cube root of the contained energy. Merging two identical toroids without energy loss therefore doubles the energy and increases both characteristic radii by a factor of approximately 1.26.

The transition to the new equilibrium is not expected to occur instantaneously. Electromagnetic fields carry energy and momentum, and their redistribution requires a finite time. Consequently, the newly formed torus may expand beyond its equilibrium dimensions, contract, and undergo transient oscillations. The redirection of the Poynting vector is investigated as a possible mechanism generating restoring electromagnetic stresses, while deviations from closed energy-flow paths may permit part of the energy to escape as radiation.

Radiation has a dual significance in the proposed model. First, it can reduce the energy associated with geometric oscillations, allowing the torus to approach equilibrium. Second, the loss of positive expansive energy can reduce the load sustained by the negative binding energy. Therefore, even if the magnitude of the binding energy remains approximately unchanged, the resulting structure may become more strongly bound relative to its expansive energy. Geometric relaxation and radiative energy loss are thus considered complementary processes rather than competing mechanisms.

The analysis combines energy conservation, geometric scaling, electromagnetic momentum transport, and a phenomenological description of damped oscillations. These relations allow the dependence of torus dimensions on energy, the conditions for local stability, and the interaction between geometric relaxation and radiation to be examined mathematically. Some coefficients, particularly those describing restoring forces, effective inertia, and radiative damping, cannot yet be evaluated numerically because they depend on the detailed nonlinear spatial geometry of the electric field and the associated distribution of electromagnetic energy flow.

The investigation focuses on merging between toroids of comparable dimensions and on the stability of the resulting toroidal configuration. Interactions between strongly dissimilar toroids are not considered. Likewise, large structural transformations that fundamentally alter the toroidal field configuration, including the possible formation of electromagnetic supertoroids, are reserved for Part A3.

The objective of Part A2 is to establish a coherent physical and mathematical

framework connecting torus merging, energy redistribution, transient oscillations, radiation, and structural stabilization. Within this framework, the article examines how an initially non-equilibrium electromagnetic structure may evolve toward a stable configuration and which conditions determine whether such stabilization is possible. [4,5,6,7]

2. Conditions for Torus Merging

2.1. Geometric Similarity

The merging of two electromagnetic toroids depends primarily on the compatibility of their spatial field configurations. Toroids with similar dimensions, field distributions, and internal dynamics can overlap more effectively than structures with substantially different geometries.

Each toroid is characterized by its major radius R, minor radius a, and the geometric ratio:

$$\beta = \frac{a}{R}$$

In the model introduced in Part A1, equilibrium toroids are assumed to maintain an approximately constant geometric ratio β and a characteristic electric-field amplitude E_0. Their dimensions depend on the electromagnetic energy contained within the structure.
For geometrically similar equilibrium toroids, the following scaling relations apply:

$$R \propto W^{1/3}$$

$$a \propto W^{1/3}$$

Consequently, comparable toroidal dimensions correspond to comparable energy contents.

The present investigation focuses on merging between toroids of similar size. Their similarity allows the electromagnetic fields to approach a common spatial configuration without requiring extensive initial restructuring. Interactions between strongly dissimilar toroids are outside the scope of this article.

2.2. Phase Alignment and Field Orientation

Geometric similarity alone does not guarantee successful merging. The electric and magnetic fields must also have compatible phases and orientations.

When the electric fields of two overlapping toroids point in similar directions, their local contributions reinforce one another. Oppositely directed fields may partially cancel. Corresponding effects arise from the magnetic-field configurations.

The interaction between two electromagnetic field configurations can be described through the cross terms in the total field energy:

$$W_{\text{int}} = \int_V \left(\varepsilon_0 E_1 \cdot E_2 + \frac{1}{\mu_0} B_1 \cdot B_2 \right) dV$$

This expression represents the interference contribution to the electromagnetic energy of the combined field. Its magnitude and sign depend on the relative phases, orientations, and spatial overlap of the interacting fields.

For identical spatial field profiles with a relative phase difference Δφ, the time-averaged interaction contribution is proportional to:

$$\langle W_{\text{int}} \rangle \propto \cos(\Delta\varphi)$$

Thus, phase alignment influences the electromagnetic interaction during overlap.

A favorable interaction energy does not, by itself, demonstrate permanent binding. Stable merging additionally requires a dynamical mechanism capable of transforming the overlapping configurations into a single persistent structure.

2.3. Spatial Overlap and Coherent Interaction

The interaction becomes most effective when corresponding regions of the electric and magnetic fields occupy approximately the same spatial positions.

In the idealized limiting case, two toroids have identical geometry, orientation, frequency, and phase. Their electromagnetic fields can then overlap completely.

The resulting electromagnetic field is described by a single combined configuration. Its mathematical description does not retain a unique decomposition into the original two toroids.

This loss of distinguishability is important. After complete overlap, the electromagnetic field need not preserve separate identities for its original contributions. Nevertheless, the disappearance of distinguishable field contributions should not be confused with proof of an irreversible physical merger.

Permanent merging requires the combined field to evolve toward a dynamically persistent configuration.

For approximately aligned but not completely overlapping toroids, only part of the electromagnetic structure participates in coherent interaction. The effectiveness of the interaction therefore depends on the spatial correspondence of the field distributions.

2.4. Duration of Coherent Interaction

A further condition concerns the duration of phase coherence.

Even toroids with similar dimensions may have slightly different internal oscillation frequencies. Their relative phase can therefore evolve during interaction.

For two frequencies ω_1 and ω_2, the phase difference evolves as:

$$\Delta\varphi(t) = \Delta\varphi(0) + (\omega_1 - \omega_2)t$$

A coherent interaction is favored when the phase difference remains sufficiently small throughout the period in which the fields overlap and redistribute energy.

This requirement introduces a relationship between the characteristic interaction time and the difference in internal frequencies.

If the relative phase changes rapidly, coherent reinforcement is reduced. If the interaction develops sufficiently quickly, moderate frequency differences may have less influence.

The actual interaction time depends on the geometry of the overlapping fields and the propagation of electromagnetic disturbances. A numerical prediction therefore requires a more detailed spatial field model.

2.5. Conditions for the Formation of a Common Structure

The preceding considerations identify three principal conditions favoring torus merging:

1. **Geometric compatibility:** the toroids have comparable radii and similar spatial field distributions.
2. **Phase and orientation compatibility:** the electric and magnetic fields permit substantial coherent interaction.
3. **Dynamical compatibility:** the combined configuration can redistribute its energy and approach a persistent equilibrium.

The first two conditions facilitate interaction. The third determines whether the interaction produces a lasting structural transformation rather than temporary field overlap.

The proposed merging mechanism assumes that sufficiently compatible toroids can form a common electromagnetic structure. This structure initially contains an energy distribution that is not necessarily compatible with its instantaneous geometry.

The subsequent evolution involves electromagnetic energy redistribution, geometric expansion, possible transient oscillations, and radiative relaxation.

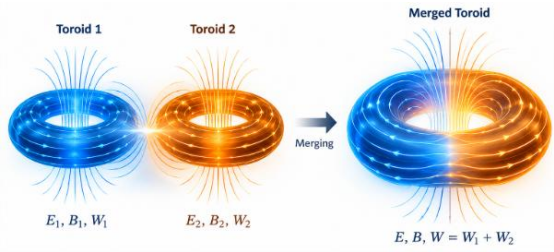


Figure 1 – Merging of Two Similar Toroids. *Two geometrically similar and phase-aligned electromagnetic toroids merge into a single higher-energy structure. Their individual field configurations are replaced by a unified electromagnetic field. The resulting torus undergoes energy redistribution and geometric expansion toward a new equilibrium, while conserving the combined energy in the absence of radiation.*

These processes are examined in the following chapters.

The central distinction is therefore between **field overlap, coherent interaction, and permanent structural merging**. The first two can be described using electromagnetic superposition and interference. The third requires the stabilization mechanism developed in the remainder of this article. [4,5,6,7]

3. Formation of a Higher-Energy Torus

3.1. From Two Toroids to a Single Electromagnetic Structure

When two electromagnetic toroids with compatible geometry, phase, and field orientation approach one another, their electromagnetic fields begin to overlap. As discussed in Chapter 2, the degree of coherent interaction depends on the spatial and temporal correspondence of their field configurations.

The proposed merging mechanism assumes that sufficiently compatible toroids may evolve from two initially independent structures into a single electromagnetic configuration.

Once complete structural merging has occurred, the resulting field no longer contains two independently identifiable toroids. Instead, it forms a common electromagnetic structure characterized by a new total energy and a corresponding spatial field distribution.

This distinction is fundamental. The merging process is not interpreted as two permanently distinguishable toroids occupying the same space. It is regarded as a transition toward a unified electromagnetic structure.

However, complete field overlap and permanent structural merging are not mathematically equivalent. The latter requires the resulting configuration to remain dynamically coherent after the initial interaction.

3.2. Energy Conservation During Merging

Consider two initially independent electromagnetic toroids with positive energies W_1 and W_2 and negative binding energies W_{v1} and W_{v2}.

In the proposed model, positive energy contributes to electromagnetic expansion, while negative binding energy represents the structural binding contribution. Both must be considered in the signed internal energy balance.

The initial energy balance is:

$$U_i = W_1 + W_2 + W_{v1} + W_{v2}$$

After merging, the resulting torus contains positive energy W_f and binding energy W_{vf}. Part of the energy may be emitted as electromagnetic radiation, denoted by W_{rad}.

Assuming no external energy exchange, conservation of the signed energy balance requires:

$$W_f + W_{vf} + W_{\text{rad}} = W_1 + W_2 + W_{v1} + W_{v2}$$

This relation accounts for possible changes in both positive and binding energy during merging.

If the total binding-energy contribution remains unchanged:

$$W_{vf} = W_{v1} + W_{v2}$$

the balance simplifies to:

$$W_f = W_1 + W_2 - W_{\mathrm{rad}}$$

For two identical toroids with initial positive energy W_0:

$$W_f = 2W_0 - W_{\mathrm{rad}}$$

In the ideal limiting case of negligible radiation:

$$W_f = 2W_0$$

Thus, two identical toroids may merge into a single structure containing twice the positive energy of either original torus, provided that the total binding-energy contribution remains unchanged and radiative losses are negligible.

If the binding energy changes during merging, its contribution must be included in the complete conservation balance.

The energy retained by the resulting torus determines its expected equilibrium dimensions under the geometric scaling assumptions adopted in this article. However, energy conservation alone does not determine the spatial distribution of the electromagnetic fields or the dynamics of the merging process.

These properties depend on the subsequent redistribution of electromagnetic energy and the establishment of a new equilibrium configuration.

3.3. Distinction Between Field Superposition and Physical Merging

An important distinction must be made between mathematical superposition and the physical process of structural merging.

For two overlapping electromagnetic field configurations, linear superposition gives:

$$\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2}$$

$$\vec{B} = \overrightarrow{B_1} + \overrightarrow{B_2}$$

The electromagnetic energy density depends quadratically on the resulting fields:

$$u_{\mathrm{EM}} = \frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2$$

Consequently, identical fields that are added coherently at the same location produce a larger instantaneous energy density than either field individually.

However, this algebraic operation does not describe the entire dynamical process by which two separate toroids approach, interact, deform, and potentially merge.

In particular, the energy associated with coherent interference must be considered together with the redistribution of the electromagnetic field and any energy or momentum exchanged during the interaction.

The physical merging process is therefore not identified with the simple addition of field amplitudes. Rather, it is treated as a dynamical reorganization of the original electromagnetic configurations into a common structure, subject to conservation of total energy.

3.4. Formation of a Non-Equilibrium Configuration

The newly merged torus initially contains more energy than either of its original components. Its geometry, however, cannot be assumed to adjust instantaneously to the increased energy.

The immediate post-merging configuration may therefore differ significantly from the equilibrium configuration corresponding to its new energy content.

In the proposed model, an equilibrium torus is characterized by a preferred electric-field amplitude E_0, an approximately constant

geometric ratio $\beta = a/R$, and dimensions determined by its energy.

If merging occurs more rapidly than geometric relaxation, the resulting structure initially contains an excess concentration of electromagnetic energy relative to its instantaneous dimensions.

The electric and magnetic fields must subsequently reorganize. Their changing spatial distribution modifies the electromagnetic stresses and the direction of energy flow.

This non-equilibrium configuration provides the initial conditions for the transient processes investigated in the following chapters.

3.5. Expected Equilibrium Geometry

For equilibrium toroids with constant geometric ratio β and characteristic energy density, the model developed in Part A1 leads to the scaling relations:

$$R \propto W^{1/3}$$

$$a \propto W^{1/3}$$

The equilibrium dimensions of a torus formed by merging two identical toroids, neglecting radiation, are therefore:

$$R_f = 2^{1/3} R_0$$
$$a_f = 2^{1/3} a_0$$

Numerically:

$$\boxed{R_f \approx 1.260 R_0, \qquad a_f \approx 1.260 a_0}$$

Type equation here.The resulting structure has twice the original volume while retaining the same geometric ratio β.

This scaling does not imply that the torus expands instantaneously or monotonically. It determines the expected equilibrium dimensions under the adopted assumptions.

During the transition, electromagnetic inertia may cause the geometry to overshoot its equilibrium dimensions and undergo transient oscillations.

The combined effects of electromagnetic restoring stresses, internal energy redistribution, and radiative losses then determine whether the structure approaches a stable configuration.

3.6. Physical Consequences of Merging

The proposed merging process has several important consequences.

First, a higher-energy electromagnetic torus may arise through the interaction of two lower-energy structures without requiring a permanent increase in the characteristic electric-field amplitude.

Second, the energy increase is accommodated primarily through geometric expansion and redistribution of the electromagnetic fields.

Third, the newly formed structure does not immediately satisfy the equilibrium conditions associated with its total energy. Its subsequent evolution involves a finite-time relaxation process.

Fourth, the interaction may generate transient oscillations and radiation. Energy emitted during relaxation reduces the energy ultimately retained in the torus and modifies its final equilibrium dimensions.

Finally, successful merging requires not only compatible initial field configurations but also the ability of the resulting structure to establish a dynamically stable electromagnetic organization.

The formation of a higher-energy torus thus connects the microscopic interaction of electromagnetic structures with their subsequent geometric and energetic evolution.

The following chapter examines this evolution quantitatively, focusing on energy

redistribution, geometric expansion, and the restoration of the preferred electromagnetic field configuration. [1,2]

4. Energy Redistribution and Geometric Expansion

4.1. Energy Concentration Following Merging

The merging of two similar electromagnetic toroids produces a structure containing more energy than either original torus. Immediately after merging, the spatial dimensions of the resulting structure may not yet correspond to its increased energy content.

For two identical toroids with initial energy W_0, the total energy in the absence of radiation is:

$$W_1 = 2W_0$$

If the combined energy initially occupies approximately the volume of one original torus, its average energy density becomes:

$$u_1 = 2u_0$$

This configuration differs from the equilibrium state described in Part A1, where the electromagnetic field amplitudes and geometric dimensions are assumed to follow a preferred structural relationship.

The increased energy concentration modifies electromagnetic stresses and initiates redistribution of the internal fields. The resulting structure can respond by expanding its geometric dimensions, thereby reducing the energy density without necessarily losing energy.

The transition from the initially concentrated configuration to the expanded equilibrium configuration constitutes the first stage of geometric relaxation.

4.2. Geometric Scaling at Constant Beta

The geometry of a torus is characterized by its major radius R and minor radius a. Its geometric volume is:

$$V = 2\pi^2 R a^2$$

The proposed equilibrium model assumes that the ratio between the radii remains constant:

$$\beta = \frac{a}{R} = \text{const.}$$

Consequently:

$$V = 2\pi^2 \beta^2 R^3$$

If the average equilibrium energy density u_0 is independent of the total energy, then:

$$W = u_0 V = 2\pi^2 \beta^2 u_0 R^3$$

The equilibrium dimensions therefore follow:

$$R(W) = R_0 \left(\frac{W}{W_0}\right)^{1/3}$$

$$a(W) = a_0 \left(\frac{W}{W_0}\right)^{1/3}$$

These relations establish a direct connection between electromagnetic energy and toroidal geometry.

They describe a family of geometrically similar equilibrium structures rather than the instantaneous geometry during a rapid interaction.

4.3. Expansion After Merging Two Identical Toroids

Consider the ideal merging of two identical toroids without radiation. The energy doubles, while the equilibrium energy density and geometric ratio remain unchanged.

Therefore:

$$\frac{V_1}{V_0} = \frac{W_1}{W_0} = 2$$

The equilibrium radii become:

$$R_1 = 2^{1/3} R_0$$

$$a_1 = 2^{1/3} a_0$$

Numerically:

$$R_1 \approx 1.260 R_0$$

$$a_1 \approx 1.260 a_0$$

Thus, the major and minor radii both increase by approximately 26%, while the volume doubles.

The expansion allows the torus to accommodate twice the original energy without requiring a permanent increase in its characteristic equilibrium field amplitude.

The initial merging configuration and the final expanded structure are therefore fundamentally different states of the same electromagnetic system.

4.4. Evolution of the Electric-Field Amplitude

The electromagnetic energy density in vacuum is:

$$u_{\mathrm{EM}} = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

To describe the geometric relaxation, consider a simplified field distribution in which the relative spatial profiles of the electric and magnetic fields remain approximately unchanged during expansion.

Under this assumption, the total electromagnetic energy can be expressed as:

$$W = C_E E_{\mathrm{amp}}^2 R a^2$$

Here, E_{amp} is the characteristic electric-field amplitude, while C_E includes the geometric field distribution and the corresponding magnetic contribution.

If the ratio β remains constant:

$$W = C_E \beta^2 E_{\mathrm{amp}}^2 R^3$$

For a fixed energy during geometric expansion:

$$E_{\mathrm{amp}}(R) \propto R^{-3/2}$$

Relative to the equilibrium configuration:

$$\frac{E_{\mathrm{amp}}(R)}{E_0} = \left(\frac{R_{\mathrm{opt}}}{R}\right)^{3/2}$$

This relation describes how the characteristic field amplitude decreases as the geometry expands.

In the idealized case of two identical toroids, if the combined energy initially occupies the original volume, the characteristic amplitude becomes:

$$E_{\mathrm{initial}} = \sqrt{2} E_0$$

As the radii increase toward their new equilibrium values, the amplitude decreases and approaches the preferred value:

$$E_{\mathrm{final}} = E_0$$

The result is consistent with the proposed structural principle that higher-energy equilibrium toroids are primarily distinguished by their dimensions rather than by permanently higher electric-field amplitudes.

This amplitude relationship is conditional on the assumed field profiles. A detailed evaluation of the spatially nonuniform electric field and the direction of electromagnetic energy flow remains necessary to establish the underlying equilibrium mechanism.

4.5. Energy Redistribution During Expansion

The expansion of the merged torus is accompanied by redistribution of electromagnetic energy throughout its volume.

The Poynting vector describes the local electromagnetic energy flux:

$$S = \frac{1}{\mu_0} E \times B$$

The electromagnetic momentum density is:

$$g = \frac{S}{c^2}$$

Changes in the geometry and orientation of the electric and magnetic fields modify the direction and spatial distribution of this energy flow.

During expansion, energy must be transferred between different regions of the toroidal

structure. These transfers are not instantaneous because changes in electromagnetic fields propagate at finite speed.

The redirection of the Poynting vector is therefore an essential candidate mechanism in the proposed geometric relaxation process.

In an ideal energy-conserving expansion, the total energy remains constant while the energy density decreases as the occupied volume increases.

For geometrically similar configurations, define the expansion factor:

$$s = \frac{R}{R_0} = \frac{a}{a_0}$$

When the total energy is twice that of the original torus:

$$\frac{u(s)}{u_0} = \frac{2}{s^3}$$

At the predicted equilibrium expansion:

$$s = 2^{1/3}$$

the original optimal energy density is restored:

$$u(s) = u_0$$

The geometric expansion thus provides an energy-conserving route from an initially concentrated electromagnetic configuration toward the assumed equilibrium energy density.

4.6. Radiative Losses and the Final Geometry

The preceding calculations describe the ideal expansion of a merged torus without radiation. During the actual merging and relaxation process, part of the electromagnetic energy may escape into the surrounding space.

In the proposed model, the positive structural energy is denoted by W, while the negative binding energy is denoted by W_v. The emitted electromagnetic energy is represented by $W_r ad$.

For two identical initial toroids, the complete signed energy balance is:

$$W_f + W_{vf} + W_{\mathrm{rad}} = 2W_0 + 2W_{v0}$$

Here, W_f and $W_v f$ represent the positive and binding energies of the resulting torus, respectively.

If the total binding-energy contribution remains unchanged during merging and relaxation:

$$W_{vf} = 2W_{v0}$$

the conservation equation simplifies to:

$$W_f = 2W_0 - W_{\mathrm{rad}}$$

Introducing the radiated fraction relative to the combined initial positive energy:

$$\eta = \frac{W_{\mathrm{rad}}}{2W_0}$$

gives:

$$W_f = 2W_0(1 - \eta)$$

For equilibrium toroids with constant geometric ratio β and constant characteristic energy density, both radii scale with the cube root of positive structural energy.

Consequently, the final equilibrium dimensions are:

$$R_f = R_0[2(1 - \eta)]^{1/3}$$

$$a_f = a_0[2(1 - \eta)]^{1/3}$$

These relations show that radiative losses reduce the final equilibrium dimensions relative to the ideal merging process without radiation.

For example, if 10% of the combined initial positive energy is emitted:

$$\eta = 0.10$$

the retained positive energy becomes:

$$W_f = 1.8W_0$$

The corresponding equilibrium radii are:

$$R_f \approx 1.216 R_0$$

$$a_f \approx 1.216 a_0$$

Thus, despite radiative losses, the resulting torus remains larger than either original torus. Its dimensions increase by approximately 21.6%, compared with 26.0% in the absence of radiation.

The radiated fraction cannot be determined from geometric scaling alone. Its numerical evaluation requires a dynamic description of the electromagnetic fields, including the deformation-induced changes in the Poynting vector and the outward energy flux.

If the binding-energy contribution also changes during merging, the simplified expressions for the retained positive energy must be replaced by the complete conservation balance. The final geometry must then be evaluated using the resulting value of W_f.

Radiation therefore influences the merging process in two related ways: it reduces the energy available for the final toroidal configuration and modifies the equilibrium dimensions toward which the structure relaxes. Its possible contribution to damping transient geometric oscillations is examined in Chapter 6.

4.7. Transition Toward Dynamic Equilibrium

The geometrically predicted equilibrium state is not necessarily reached through monotonic expansion.

Because electromagnetic fields carry momentum, their redistribution can involve inertial effects. The torus may expand beyond its equilibrium dimensions before the direction of geometric adjustment reverses.

This behavior introduces the possibility of transient oscillations around an energy-dependent equilibrium geometry.

At the same time, radiation may remove part of the electromagnetic energy, causing the equilibrium dimensions themselves to change during the relaxation process.

The transition therefore involves two coupled effects: geometric adjustment toward the instantaneous equilibrium configuration and gradual modification of that equilibrium through energy loss.

The equations developed in this chapter establish the relationship between energy content, field amplitude, and equilibrium geometry. They also provide the initial conditions for examining electromagnetic restoring mechanisms and transient oscillations.

The following chapter investigates how changes in the direction of the Poynting vector can contribute to electromagnetic stresses and geometric stabilization. [1,2]

5. Poynting Vector Redirection

5.1. Electromagnetic Energy Flow in a Toroidal Structure

The geometric stabilization of an electromagnetic torus requires a mechanism capable of responding to changes in its dimensions. In the proposed model, an important role is assigned to the direction and curvature of electromagnetic energy flow.

The local energy flux is described by the Poynting vector:

$$S = \frac{1}{\mu_0} E \times B$$

The associated electromagnetic momentum density is:

$$g = \frac{S}{c^2}$$

In the toroidal model introduced in Part A1, electromagnetic energy is assumed to circulate along predominantly closed trajectories. The electric and magnetic field configurations determine the local orientation of the Poynting

vector and the spatial organization of the energy flow.

A stable toroidal structure requires this energy transport to remain compatible with its geometry. If the torus is deformed, the field distribution and the direction of energy flow must adjust accordingly.

Because electromagnetic energy carries momentum, changing the direction of its transport requires a corresponding redistribution of momentum. This provides a possible connection between the geometry of the torus and the electromagnetic stresses acting within it.

5.2. Momentum Change Along a Curved Energy Path

Consider a small region in which electromagnetic energy propagates predominantly along a curved trajectory with local radius of curvature ρ.

For an idealized locally propagating electromagnetic wave, the momentum transported per unit energy is 1/c.

If a power P changes its direction through an angle θ, the magnitude of the corresponding momentum-transfer rate is:

$$F = \frac{2P}{c}\sin\left(\frac{\theta}{2}\right)$$

For a small angular change:

$$F \approx \frac{P}{c}$$

Along a curved path, the change in direction over an arc length dl is:

$$d\theta = \frac{dl}{\rho}$$

For energy locally transported at speed c, the resulting characteristic transverse force density is:

$$f_S \approx \frac{u}{\rho}$$

Here, u represents the electromagnetic energy density associated with the propagating contribution.

This relation expresses the momentum-transfer scale associated with directional changes in energy transport. It does not independently determine the net force acting on a toroidal structure, because the complete electromagnetic stress distribution must also be considered.

Nevertheless, it provides a useful quantitative measure of the sensitivity of internal momentum redirection to geometric deformation.

5.3. Geometric Dependence of the Redirection Effect

To investigate this sensitivity, consider a geometrically similar deformation of a torus at constant total electromagnetic energy.

The geometric ratio remains fixed:

$$\beta = \frac{a}{R} = \text{const.}$$

Introduce the relative geometric scale:

$$s = \frac{R}{R_0} = \frac{a}{a_0}$$

Since the toroidal volume is proportional to the third power of its linear dimensions, the average energy density follows:

$$\frac{u}{u_0} = s^{-3}$$

Assume that the characteristic curvature radius of the internal energy-flow trajectories scales with the torus dimensions:

$$\rho = s\rho_0$$

Substitution into the momentum-redirection relation gives:

$$\frac{f_S}{f_{S0}} = s^{-4}$$

Thus, the characteristic local momentum-redirection force density varies with the fourth power of the inverse geometric scale.

This result is significant because it indicates that the redirection effect is more sensitive to deformation than the average energy density.

A contraction increases both quantities, but the characteristic redirection force density increases more rapidly. An expansion reduces both quantities, with the redirection effect decreasing more rapidly.

The relation is conditional on geometrically similar energy-flow trajectories and on the local traveling-wave approximation. Its coefficient and directional contribution to the actual restoring stress depend on the spatial field structure.

5.4. Response to Small Geometric Deformations

For small deviations from equilibrium, write:

$$s = 1 + \delta$$

where δ represents the relative geometric deformation.

Expanding the energy-density relation to first order gives:

$$\frac{u}{u_0} \approx 1 - 3\delta$$

For the momentum-redirection force density:

$$\frac{f_S}{f_{S0}} \approx 1 - 4\delta$$

A contraction of 1% therefore increases the average energy density by approximately 3% and the characteristic redirection force density by approximately 4%.

Similarly, a 1% expansion reduces the corresponding quantities by approximately 3% and 4%.

The exact nonlinear scaling relations give approximately 3.1% and 4.1% increases for a 1% contraction.

These differences provide a possible basis for a geometric feedback mechanism.

However, the stronger variation of the redirection effect does not by itself demonstrate stability. A restoring mechanism requires that the resulting electromagnetic force act against the imposed deformation.

The necessary relationship between momentum redirection, field orientation, and net restoring force must therefore be examined separately.

5.5. Electromagnetic Stresses and the Restoring Mechanism

The complete electromagnetic momentum balance is expressed through Maxwell's stress tensor.

In vacuum, its components are:

$$T_{ij} = \varepsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij} E^2 \right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2 \right)$$

The local momentum balance can be written as:

$$f = \nabla \cdot T - \frac{\partial g}{\partial t}$$

Here, f is the electromagnetic force density on matter or other explicitly modeled interacting degrees of freedom. In a source-free vacuum, the right-hand side vanishes, although electromagnetic momentum can still be redistributed through the stress flux.

The stress tensor therefore provides the appropriate framework for determining how electromagnetic momentum is transmitted through different regions of a toroidal configuration.

In the proposed model, the nonuniform electric-field geometry and the associated magnetic-field organization may generate competing contributions to the deformation response.

A contraction changes the field concentration and curvature of the energy-flow trajectories. An expansion produces the opposite geometric change.

For local stability, the combined electromagnetic response must reverse its effective radial direction on opposite sides of equilibrium.

If the resultant radial force is denoted by F_R, the equilibrium condition is:

$$F_R(R_0) = 0$$

The corresponding local restoring condition is:

$$\left.\frac{dF_R}{dR}\right|_{R_0} < 0$$

This condition identifies a stable radial response for the selected deformation mode, provided its effective inertia is positive.

The quantitative evaluation of the derivative requires the full spatial structure of the electric and magnetic fields, including their changing directions during deformation.

In particular, the electric field proposed in Part A1 has a strongly nonuniform spatial geometry. Its contribution to the stress distribution cannot be accurately represented by a uniform-field approximation.

Consequently, the present analysis establishes the necessary stability condition and the geometric sensitivity of momentum redirection, but does not yet determine the numerical restoring-force coefficient.

5.6. Deformation-Induced Energy Leakage

The redirection of the Poynting vector may also influence the ability of the torus to retain electromagnetic energy.

In an idealized closed energy-flow configuration, the net outward energy flux through the boundary of the structure is zero.

For a fixed volume in a source-free region, Poynting's theorem gives:

$$\frac{dW}{dt} = -\oint_{\partial V} \mathrm{b}S \cdot n\, dA$$

Here, $\vec{n}$ is the outward surface normal.

A geometric deformation can modify the orientation of the local energy flow. If part of the Poynting vector acquires an outward component that is not compensated by returning energy flow, electromagnetic energy can escape from the toroidal structure.

For a local deflection angle α relative to the tangential direction:

$$S_n = \mathrm{S}\sin\alpha$$

For small deflections:

$$S_n \approx S\alpha$$

The actual radiated power depends on the integrated outward flux and the time-dependent field distribution.

A local outward component is not necessarily equivalent to irreversible radiation, because energy may subsequently return to the structure. For a moving toroidal boundary, the energy carried across the changing boundary must also be included in the balance.

The model therefore distinguishes two related consequences of Poynting-vector redirection.

The first is electromagnetic momentum redistribution, which may contribute to restoring forces.

The second is energy leakage, which can remove energy from a disturbed toroidal configuration.

These processes may act simultaneously during geometric relaxation.

5.7. Role in Toroidal Stabilization

The mathematical relations developed above identify a potentially important property of toroidal electromagnetic structures: the characteristic momentum-redirection effect responds more strongly to geometrically

similar deformation than the average electromagnetic energy density.

At constant total energy, the respective dependencies are:

$$u \propto R^{-3}$$

$$f_S \propto R^{-4}$$

This difference provides a quantitative basis for investigating the response of the internal energy-flow geometry to compression and expansion.

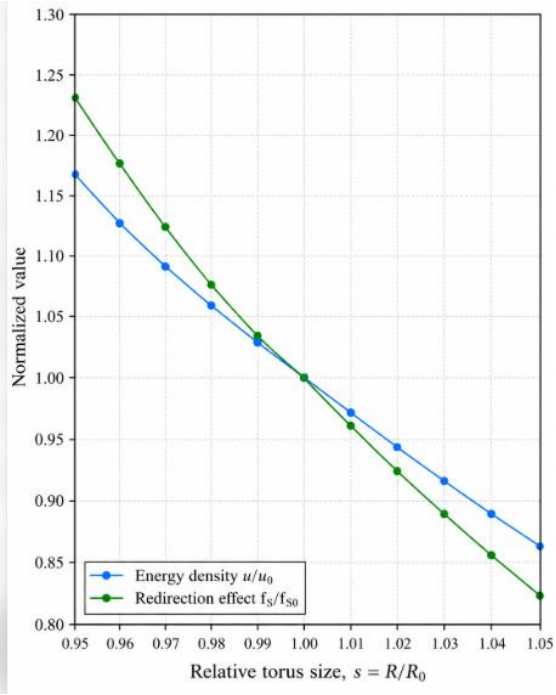


Figure 2 – Response of a Toroid to Geometric Deformation. *The diagram compares the normalized electromagnetic energy density and the local momentum-redirection effect during torus contraction and expansion. Energy density varies as s^{-3}, while the redirection effect varies as s^{-4}. The stronger sensitivity of momentum redirection to geometric changes provides a potential mechanism for restoring forces and toroidal stabilization.*

The proposed stabilization mechanism requires an appropriate balance between electromagnetic stresses, together with a restoring response that changes direction when the equilibrium geometry is crossed.

Because electromagnetic fields carry momentum, the geometric response need not be instantaneous. A disturbed torus may overshoot its equilibrium dimensions and develop transient oscillations.

If such oscillations modify the closed energy-flow paths, part of their energy may escape through radiation. The combined action of restoring stresses and radiative losses can then produce a damped relaxation process.

The principal limitation remains the detailed nonlinear spatial geometry of the electric field and its coupling to the magnetic field. Without this information, the effective restoring stiffness and radiative damping coefficient cannot yet be evaluated numerically.

The following chapter develops a phenomenological description of transient oscillations and radiation, using the geometric and energetic relations established here. [1,3,7]

6. Transient Oscillations and Radiation

6.1. Electromagnetic Inertia During Geometric Relaxation

The merging of two electromagnetic toroids produces a structure whose initial geometry may differ from the equilibrium configuration corresponding to its increased energy. As discussed in the preceding chapters, the redistribution of electromagnetic energy modifies the internal field geometry and the direction of the Poynting vector.

Electromagnetic fields carry both energy and momentum. Consequently, changes in their spatial distribution involve momentum transport and cannot occur instantaneously.

This introduces an inertial response into the geometric relaxation process. When the torus expands toward its equilibrium dimensions, the redistribution of electromagnetic

momentum may continue beyond the equilibrium configuration. The resulting structure can therefore overshoot its optimal dimensions and subsequently contract.

Such behavior provides the basis for transient geometric oscillations.

To describe this process, an effective inertia parameter is introduced:

$$M_{\text{eff}} = \alpha \frac{W}{c^2}$$

Here, W is the electromagnetic energy associated with the structure, while α represents the participation of the electromagnetic momentum distribution in the selected deformation mode.

The coefficient α depends on the spatial organization of the fields and is not assumed to be universal. The relation is therefore an effective model of geometric inertia rather than a complete derivation from Maxwell's equations.

6.2. Oscillations Around the Equilibrium Geometry

For a given retained energy W, the equilibrium major radius is determined by the geometric scaling law:

$$R_{\text{opt}}(W) = R_0 \left(\frac{W}{W_0}\right)^{1/3}$$

The corresponding minor radius follows:

$$a_{\text{opt}}(W) = \beta R_{\text{opt}}(W)$$

Let the deviation from the instantaneous equilibrium radius be:

$$y(t) = R(t) - R_{\text{opt}}\big(W(t)\big)$$

For sufficiently small geometric deviations, the restoring force can be approximated by:

$$F_R = -Ky$$

The coefficient K represents the effective radial stiffness of the electromagnetic structure.

If K>0, the restoring force acts toward equilibrium. If K<0, small radial deviations increase rather than diminish, indicating local instability.

For approximately constant energy and negligible damping, the resulting equation of motion is:

$$M_{\text{eff}}\ddot{y} + Ky = 0$$

Its natural angular frequency is:

$$\omega_0 = \sqrt{\frac{K}{M_{\text{eff}}}}$$

The corresponding oscillation period is:

$$T_0 = 2\pi\sqrt{\frac{M_{\text{eff}}}{K}}$$

These expressions demonstrate that the characteristic oscillation time depends on the balance between effective electromagnetic inertia and restoring stiffness.

The numerical values of M_{eff} and K require knowledge of the detailed electric- and magnetic-field geometry, particularly the response of the Poynting-vector distribution to deformation.

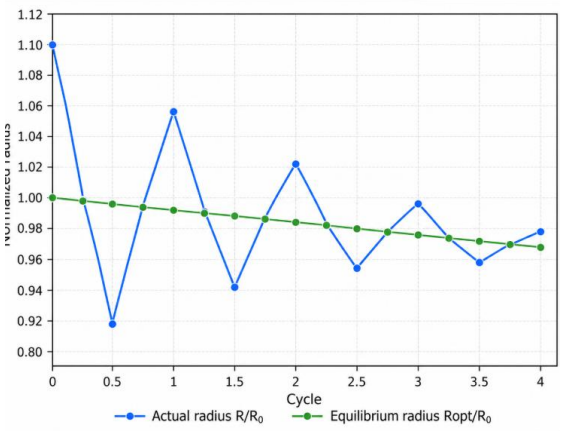


Figure 3 – Transient Oscillations Around a Moving Equilibrium. *Illustrative evolution of the torus radius during radiative relaxation. Electromagnetic inertia permits geometric overshoot and damped oscillations, while the gradual loss of retained energy shifts the*

equilibrium radius. The curves demonstrate the proposed coupled behavior rather than a numerical solution of the complete electromagnetic field equations.

6.3. Radiation as a Damping Mechanism

During geometric oscillation, the electric and magnetic fields continuously change their spatial distribution. Their associated energy flow may deviate from the closed trajectories characteristic of the equilibrium torus.

If part of this energy flow escapes into the surrounding space, the torus emits electromagnetic radiation.

Radiation removes energy from the oscillating structure and may reduce the amplitude of its geometric motion.

For small oscillations, this effect can be represented phenomenologically by a damping term proportional to the deformation velocity:

$$F_{\text{damp}} = -\Gamma \dot{y}$$

The corresponding equation becomes:

$$M_{\text{eff}} \ddot{y} + \Gamma \dot{y} + K y = 0$$

Here, Γ is the effective damping coefficient.

The condition for oscillatory relaxation is:

$$0 < \Gamma < 2\sqrt{K M_{\text{eff}}}$$

Define the damping ratio:

$$\zeta = \frac{\Gamma}{2\sqrt{K M_{\text{eff}}}}$$

For $0 < \zeta < 1$, the approximate solution is:

$$y(t) = A_0 e^{-\zeta \omega_0 t} \cos(\omega_d t + \varphi)$$

where:

$$\omega_d = \omega_{0\sqrt{1-\zeta^2}}$$

Thus, a disturbed torus may perform several oscillations while gradually reducing their amplitude.

The damping coefficient is not known numerically because radiation depends on the time-dependent spatial geometry of the electromagnetic fields and on the portion of the Poynting flux that escapes the structure.

The linear damping model describes the expected form of the response near equilibrium. It does not imply that electromagnetic radiation necessarily produces a velocity-proportional force under arbitrary deformation.

6.4. Energy Loss During Oscillations

For constant effective inertia and stiffness, the mechanical analogue of the geometric oscillation energy is:

$$W_{\text{osc}} = \frac{1}{2} M_{\text{eff}} \dot{y}^2 + \frac{1}{2} K y^2$$

Differentiating and applying the damped oscillator equation gives:

$$\frac{dW_{\text{osc}}}{dt} = -\Gamma \dot{y}^2$$

The corresponding damping power is:

$$P_{\text{osc}} = \Gamma \dot{y}^2$$

If damping is caused entirely by electromagnetic radiation, this energy leaves the torus as radiated electromagnetic energy.

The decreasing oscillation energy causes the amplitude of geometric deformation to diminish.

For weak damping, the amplitude reduction after one oscillation period is approximately:

$$\frac{A_{n+1}}{A_n} = e^{-2\pi\zeta/\sqrt{1-\zeta^2}}$$

The corresponding oscillation-energy ratio is:

$$\frac{W_{\text{osc},n+1}}{W_{\text{osc},n}} = e^{-4\pi\zeta/\sqrt{1-\zeta^2}}$$

These expressions quantify the relaxation process without requiring a numerical value for the damping coefficient.

They also show that the energy of geometric oscillation decreases more rapidly than its amplitude.

6.5. Loss of Positive Structural Energy

Radiation may influence not only the energy of geometric oscillations but also the positive structural energy retained within the electromagnetic torus.

The proposed model distinguishes positive structural energy W from negative binding energy W_v. Their signed internal energy balance is:

$$U = W + W_v$$

During radiative relaxation, part of the positive energy may escape, while the binding-energy contribution remains approximately unchanged over the same interval.

Under this assumption:

$$W_f = W_i - W_{\mathrm{rad}}$$

$$W_{vf} \approx W_{vi}$$

The relative expansive loading is described by:

$$q = \frac{W}{|W_v|}$$

If positive energy decreases while the magnitude of binding energy remains constant:

$$q_f = \frac{W_i - W_{\mathrm{rad}}}{|W_{vi}|} < q_i$$

Consequently, the same binding contribution sustains a smaller positive expansive energy. Its relative dominance increases without requiring an increase in the absolute magnitude of negative binding energy.

Within the proposed model, this represents a reduction in expansive loading and a shift toward stronger relative binding dominance.

This energetic change must be distinguished from geometric stability. A lower value of q does not by itself establish a restoring force or guarantee stability against deformation.

Furthermore, the energy associated with geometric oscillations and the positive structural energy are not independent sources of radiated energy. They must be included consistently in the complete conservation balance to avoid counting the same energy loss twice.

The detailed consequences of this energy redistribution for structural binding and stability are examined in Chapter 7.

6.6. Coupled Evolution of Energy and Geometry

Energy loss also modifies the equilibrium dimensions of the torus.

For the family of equilibrium configurations defined by constant β and constant characteristic energy density:

$$R_{\mathrm{opt}}(W) = R_0 \left(\frac{W}{W_0}\right)^{1/3}$$

Differentiation gives:

$$\frac{\dot{R_{\mathrm{opt}}}}{R_{\mathrm{opt}}} = \frac{1}{3}\frac{\dot{W}}{W}$$

Thus, decreasing retained energy corresponds to decreasing equilibrium dimensions.

The actual torus radius can be represented as:

$$R(t) = R_{\mathrm{opt}}\big(W(t)\big) + y(t)$$

This expression separates two simultaneous processes.

The first is the gradual change of the equilibrium radius as the torus loses energy.

The second is the geometric oscillation around that changing equilibrium radius.

If the equilibrium radius changes slowly compared with the oscillation period, the damped harmonic approximation remains applicable locally.

If energy loss becomes rapid, the moving equilibrium must be included explicitly in the dynamical equation. In that regime, the

restoring-force equation for the physical radius takes the approximate form:

$$M_{\mathrm{eff}}\ddot{R} + \Gamma\dot{R} + K\left(R - R_{\mathrm{opt}}(W)\right) = 0$$

The coefficients may themselves vary with energy and geometry.

This coupled description is important because radiation does not merely damp geometric motion. It can also alter the location of the equilibrium toward which the structure is evolving.

6.7. Conditions for Successful Relaxation

The relaxation process can lead to a stable configuration when the restoring mechanism remains effective and radiative losses do not destroy the underlying field organization.

For small perturbations, the primary conditions are positive effective inertia, positive restoring stiffness, and nonnegative damping.

The structure may then approach its energy-dependent equilibrium through damped oscillations.

At larger deformations, however, the spatial field distribution may depart substantially from the regime in which linear restoring forces and damping remain valid.

Changes in the topology and curvature of energy-flow paths may then modify the restoring response and the ability of the torus to retain electromagnetic energy.

The present model does not assign a universal critical deformation to this transition. Its determination requires the detailed nonlinear spatial geometry of the electric field and the corresponding electromagnetic momentum balance.

An additional possibility is that energy loss changes the torus into a different, but still bound, electromagnetic configuration rather than causing complete disintegration. Large structural transformations of this type are considered in Part A3.

6.8. Physical Interpretation

Transient oscillations and radiation provide a possible connection between the formation of a higher-energy torus and its subsequent stabilization.

The proposed process begins with an electromagnetic structure whose energy distribution is not compatible with its instantaneous geometry.

Restoring electromagnetic stresses drive the structure toward equilibrium, while the inertia associated with electromagnetic momentum transport may produce overshoot and oscillation.

Radiative losses can then reduce the oscillation energy and, under the assumed energy partition, decrease the positive expansive energy relative to the negative binding energy.

The torus may consequently approach a new equilibrium with smaller geometric oscillations and reduced expansive loading.

The mathematical framework establishes the conditions under which such relaxation is possible. Its quantitative confirmation requires the spatial field distributions needed to determine the effective inertia, restoring stiffness, and radiation rate.

The following chapter examines the energetic consequences of this process in greater detail, particularly the relationship between positive energy loss, negative binding energy, and increasing structural stability. [1,3,8]

7. Binding Energy and Increasing Stability

7.1. Positive Energy and Negative Binding Energy

The proposed electromagnetic torus model distinguishes two contributions to its internal energy organization: positive structural energy and negative binding energy.

Positive energy, denoted by W, is associated with the electromagnetic field configuration and its expansive effects. Negative binding energy, denoted by W_v, represents the energetic contribution associated with structural confinement.

Their signs are defined as:

$$W > 0, \qquad W_v < 0$$

The signed internal energy balance is expressed as:

$$U = W + W_v$$

Within the proposed model, a negative value of U indicates that the magnitude of the binding contribution exceeds the positive structural energy:

$$U < 0 \quad \Leftrightarrow \quad |W_v| > W$$

This condition characterizes a binding-dominated energy balance.

The negative binding energy must be distinguished from conventional electromagnetic field energy. In Maxwell's electrodynamics, both electric and magnetic field energies are positive. Here, W_v represents a proposed structural binding contribution defined relative to a chosen reference configuration.

The signed energy balance also differs from the model's description of inertia, in which the magnitudes of the positive and binding contributions may both participate. Consequently, the energy balance U should not be identified directly with the inertial energy of the structure without an additional physical derivation.

The distinction between W and W_v becomes particularly important when radiation removes energy from the torus. If positive structural energy decreases while the magnitude of binding energy remains approximately unchanged, the structure becomes more strongly binding-dominated in relative terms.

This change in energetic balance does not automatically establish dynamical stability. Stable confinement additionally requires an electromagnetic restoring mechanism capable of opposing geometric deformation.

The following sections examine how radiative energy loss modifies the relationship between positive structural energy and negative binding energy, and how this change may contribute to the stabilization of the resulting torus.

7.2. Reduction of Expansive Energy

During transient relaxation, an electromagnetic torus may lose part of its positive structural energy through radiation.

Consider a torus with initial positive energy W_i and negative binding energy $W_v i$. Its initial signed internal energy balance is:

$$U_i = W_i + W_{vi}$$

Let $W_r ad$ denote the positive energy emitted as electromagnetic radiation.

If the binding-energy contribution remains approximately unchanged during this process:

$$W_{vf} \approx W_{vi}$$

conservation of the signed energy balance gives:

$$W_f = W_i - W_{\mathrm{rad}}$$

The final internal energy balance is therefore:

$$U_f = W_f + W_{vf}$$

Substituting the preceding relations gives:

$$U_f \approx U_i - W_{\mathrm{rad}}$$

Thus, emission of positive energy makes the signed internal energy balance more negative, provided that the binding-energy contribution remains approximately constant.

The absolute magnitude of negative binding energy has not increased. Instead, the same binding contribution is associated with a smaller positive expansive energy.

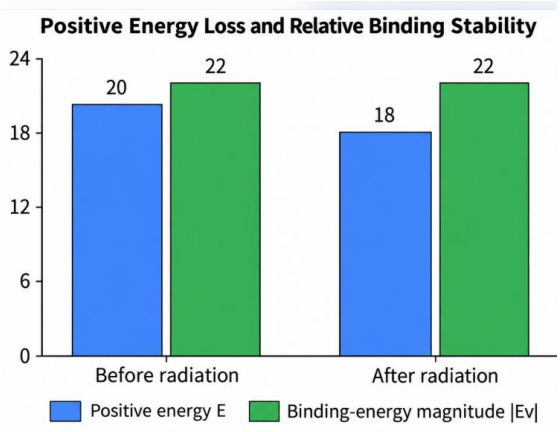


Figure 4 – Positive Energy Loss and Relative Binding Stability. *The diagram illustrates the reduction of positive expansive energy from 20 to 18 units while the magnitude of negative binding energy remains constant at 22 units. The relative expansive loading decreases, indicating stronger binding dominance without an increase in absolute binding energy. The values are illustrative and do not represent a numerically calculated toroidal configuration.*

Within the proposed model, this represents an increase in relative binding dominance without requiring additional binding energy.

The result is conditional on the assumed separation between positive structural energy and negative binding energy. If the binding contribution changes during radiation, its variation must also be included in the conservation balance.

A more negative internal energy balance does not by itself demonstrate greater resistance to geometric deformation. Dynamical stability continues to depend on the electromagnetic restoring response.

The following section introduces a dimensionless ratio that quantifies the changing relationship between positive structural energy and negative binding energy.

7.3. Relative Binding Stability

To quantify the relationship between positive structural energy and negative binding energy, define the dimensionless ratio:

$$q = \frac{W}{|W_v|}$$

This quantity describes the relative expansive loading of the binding structure.

Three regimes can be distinguished:

- For q > 1, positive structural energy exceeds the magnitude of the binding contribution.
- For q = 1, the two contributions have equal magnitudes.
- For q < 1, the binding contribution dominates the signed internal energy balance.

The proposed model associates a decreasing value of q with reduced expansive loading and greater relative binding dominance.

For constant binding energy, differentiation gives:

$$dq = \frac{dW}{|W_v|}$$

If positive structural energy decreases:

$$dW<0$$

then:

$$dq<0$$

Thus, radiative reduction of positive energy directly decreases the relative loading of the binding structure, provided that the magnitude of negative binding energy remains unchanged.

More generally, when both energy contributions change, the differential relation becomes:

$$\frac{dq}{q} = \frac{dW}{W} - \frac{d|W_v|}{|W_v|}$$

This relation shows that relative binding dominance depends on the changes in both energy contributions. A decrease in positive energy is therefore not the only possible mechanism for reducing q.

The ratio q provides a measure of energetic binding dominance within the proposed model. However, it does not independently determine the restoring force or guarantee stability against geometric deformation.

Dynamical stability additionally requires that the electromagnetic stresses oppose departures from equilibrium, as examined in Chapter 8.

7.4. Numerical Illustration

Consider a toroidal configuration with the following initial energy contributions:

$$W_i = 20, \qquad W_{vi} = -22$$

The initial signed internal energy balance is:

$$U_i = W_i + W_{vi} = 20 - 22 = -2$$

The corresponding relative expansive loading is:

$$q_i = \frac{W_i}{|W_{vi}|} = \frac{20}{22} \approx 0.909$$

Suppose the torus emits two units of positive energy while its binding-energy contribution remains unchanged.

The resulting configuration has:

$$W_f = 18, \qquad W_{vf} = -22$$

The final signed internal energy balance becomes:

$$U_f = W_f + W_{vf} = 18 - 22 = -4$$

The relative expansive loading decreases to:

$$q_f = \frac{W_f}{|W_{vf}|} = \frac{18}{22} \approx 0.818$$

The positive structural energy has decreased by 10%, while the magnitude of negative binding energy remains constant. Consequently, the relative expansive loading decreases from approximately 90.9% to 81.8%.

The energy balance is consistent with the emission of two energy units:

$$U_i = U_f + W_{\mathrm{rad}} = -4 + 2 = -2$$

The result illustrates a central feature of the proposed model: increased relative binding dominance does not require an increase in the absolute magnitude of negative binding energy.

The numerical values are illustrative and do not represent a calculated electromagnetic torus. A more negative internal energy balance indicates stronger energetic binding dominance within the model, while dynamical stability must be established independently through the electromagnetic restoring response.

7.5. Radiation and the Stabilization Process

The preceding chapter examined radiation as a mechanism capable of damping transient geometric oscillations.

The present energy balance introduces a complementary consequence: radiation may reduce the positive structural energy that contributes to expansive loading.

These two effects are related but should not be confused.

The reduction of oscillation energy decreases the amplitude of transient geometric motion. The reduction of positive structural energy changes the relationship between expansion and binding.

In a complete electromagnetic model, both effects must be included in a single conservation balance. Their separation is useful for describing the relaxation process, provided that emitted energy is not counted twice.

The resulting stabilization mechanism may therefore involve two simultaneous developments.

First, the torus approaches a geometrically admissible equilibrium configuration.

Second, its positive expansive energy decreases relative to the retained binding contribution.

These processes can reinforce one another. Radiation removes energy associated with departure from equilibrium while potentially reducing the expansive loading of the final structure.

However, a decrease in positive energy does not automatically guarantee that every deformation becomes more stable. The electromagnetic restoring response and the persistence of the binding configuration remain essential.

7.6. Energy Loss and Equilibrium Geometry

A change in the positive structural energy retained by a torus also affects its equilibrium dimensions.

For the family of geometrically similar equilibrium toroids considered in this article, the geometric ratio remains constant:

$$\beta = \frac{a}{R} = \text{const}$$

Assuming that the average positive structural energy density is independent of the total energy, the equilibrium dimensions scale with the cube root of W:

$$R_{\text{opt}} \propto W^{1/3}$$

$$a_{\text{opt}} \propto W^{1/3}$$

Let W_i denote the initial positive structural energy and R_i and a_i the corresponding equilibrium radii.

After a change in positive energy, the new equilibrium dimensions are:

$$R_{\text{opt}} = R_i \left(\frac{W}{W_i}\right)^{1/3}$$

$$a_{\text{opt}} = a_i \left(\frac{W}{W_i}\right)^{1/3}$$

If the torus emits electromagnetic radiation while its binding-energy contribution remains approximately unchanged:

$$W = W_i - W_{\text{rad}}$$

the equilibrium dimensions become:

$$R_{\text{opt}} = R_i \left(1 - \frac{W_{\text{rad}}}{W_i}\right)^{1/3}$$

$$a_{\text{opt}} = a_i \left(1 - \frac{W_{\text{rad}}}{W_i}\right)^{1/3}$$

Thus, a reduction in retained positive structural energy corresponds to smaller equilibrium dimensions within the assumed family of configurations.

For small changes in energy, differentiation gives:

$$\frac{dR_{\text{opt}}}{R_{\text{opt}}} = \frac{1}{3}\frac{dW}{W}$$

The geometric response, however, need not be instantaneous. Electromagnetic momentum redistribution may produce transient oscillations around the changing equilibrium radius, while radiation gradually removes energy from the structure.

The resulting torus may therefore approach a configuration with smaller equilibrium dimensions, reduced oscillation amplitude, and lower relative expansive loading.

The binding-energy contribution need not become more negative during this process. However, if it changes significantly, the complete energy balance developed in Chapter 3 must be applied.

These geometric relations are conditional on the equilibrium family retaining approximately constant geometric ratio and characteristic positive energy density. Substantial changes in the internal electromagnetic field geometry may lead to departures from the proposed scaling.

7.7. Energetic Stability and Structural Persistence

The distinction between energetic binding dominance and dynamical stability is essential.

A torus may have a strongly negative signed internal energy balance while still possessing a deformation mode that grows with time.

Conversely, a configuration with relatively small binding dominance may remain locally stable if its electromagnetic stresses provide sufficiently strong restoring forces.

The model therefore distinguishes three related properties.

Energetic binding dominance describes the relative magnitude of positive and negative energy contributions.

Local geometric stability describes the response to small changes in torus dimensions and field configuration.

Structural persistence describes the ability of the electromagnetic configuration to remain organized over extended periods, including after finite disturbances.

Radiative energy loss can improve the first property and can contribute to the second by damping oscillations. The third depends on the continued existence of a coherent and dynamically stable field configuration.

The proposed model allows these properties to evolve together without requiring them to be identical.

7.8. Implications for Long-Term Stability

The interaction between positive energy loss and persistent negative binding energy suggests a possible mechanism for the gradual stabilization of electromagnetic toroidal structures.

After merging, a torus may initially contain excess positive energy relative to its instantaneous geometry. The resulting electromagnetic stresses cause geometric adjustment and possible oscillations.

During relaxation, radiation can reduce both transient excitation and the positive energy retained within the structure.

If the binding contribution remains approximately unchanged, its relative dominance increases.

The torus may consequently settle into a configuration that is less strongly driven toward expansion and more resistant to further energetic disturbance.

This mechanism also suggests that a stable electromagnetic structure need not correspond to a configuration in which all radiative losses have been absent. Limited radiation may instead be an integral part of its formation and relaxation.

Nevertheless, the final stability depends on the existence of an appropriate electromagnetic restoring mechanism and the compatibility of the retained energy with the geometry of the structure.

The energy relationships presented here establish a mathematical framework for evaluating binding dominance. The following chapter examines whether the corresponding electromagnetic configuration is also locally stable under geometric deformation. [1,3,8]

8. Local Stability and Stress Tests

8.1. Definition of Local Stability

The stability of an electromagnetic torus depends on its response to disturbances of the equilibrium field configuration. A stable structure must be capable of limiting small geometric deviations and, under dissipative conditions, returning toward an equilibrium state.

In the proposed model, the equilibrium geometry is characterized by the major radius R_0, minor radius a_0, and geometric ratio:

$$\beta = \frac{a_0}{R_0}$$

For the initial analysis, both radii are assumed to change proportionally, preserving the geometric similarity of the torus.

Introduce the relative deformation:

$$s = \frac{R}{R_0} = \frac{a}{a_0}$$

The equilibrium state corresponds to:

$$s=1$$

A contraction corresponds to s<1, while an expansion corresponds to s>1.

Local stability requires that the electromagnetic response oppose sufficiently small deviations from equilibrium.

This condition can be investigated through the restoring-force response, the energy associated with deformation, and the resulting oscillatory dynamics.

8.2. Energy Density Under Geometric Deformation

The geometric volume of a torus is:

$$V = 2\pi^2 R a^2$$

For proportional changes in R and a, the volume scales as:

$$V(s) = V_0 s^3$$

If the total electromagnetic energy remains constant during a sufficiently short deformation interval, the average energy density becomes:

$$u(s) = u_0 s^{-3}$$

A contraction therefore increases the energy density, while an expansion decreases it.

For small relative deformation:

$$s = 1 + \delta$$

the first-order approximation is:

$$\frac{u}{u_0} \approx 1 - 3$$

Thus, a 1% geometric contraction increases the average energy density by approximately 3%, whereas a 1% expansion decreases it by approximately 3%.

This sensitivity provides a direct relationship between geometric deformation and electromagnetic energy concentration.

8.3. Response of Poynting-Vector Redirection

As established in Chapter 5, the characteristic local momentum-redirection force density associated with a curved electromagnetic energy flow can be approximated by:

$$f_S \approx \frac{u}{\rho}$$

Here, ρ represents the local radius of curvature of the energy-flow trajectory.

If the characteristic curvature radius changes proportionally with the torus dimensions:

$$\rho(s) = s\rho_0$$

then:

$$\frac{f_S}{f_{S0}} = s^{-4}$$

For small deformations:

$$\frac{f_S}{f_{S0}} \approx 1 - 4$$

The fourth-power dependence indicates a stronger geometric sensitivity than that of the average energy density.

A relatively small contraction produces a noticeable increase in the characteristic momentum-redirection effect. Expansion produces a corresponding decrease.

However, this calculation gives the magnitude of a local momentum-transfer scale, not the resultant force responsible for restoring the torus.

The direction and effectiveness of the restoring response depend on the full spatial electric- and magnetic-field configuration.

8.4. Numerical Stress Tests

Two symmetric geometric deformation amplitudes are considered: 1% and 5%.

The calculations use the exact scaling relations:

$$\frac{u}{u_0} = s^{-3}$$

$$\frac{f_S}{f_{S0}} = s^{-4}$$

The resulting normalized values are:

Geometric deformation	Relative radius (s)	Energy density u/u_0	Redirection effect f_S/f_{S0}
5% contraction	0.95	1.166	1.228
1% contraction	0.99	1.031	1.041
Equilibrium	1.00	1.000	1.000
1% expansion	1.01	0.971	0.961
5% expansion	1.05	0.864	0.823

Table 1 – Numerical Results of Geometric Stress Tests. *The table compares the normalized electromagnetic energy density and characteristic momentum-redirection force density for toroidal contractions and expansions of 1% and 5%. The results demonstrate the greater sensitivity of momentum redirection to geometric deformation, as predicted by the inverse-fourth-power scaling relation.*

The results show that a 5% contraction increases the characteristic momentum-redirection force density by approximately 23%, while a 5% expansion reduces it by approximately 18%.

The corresponding changes in average energy density are approximately +17% and −14%.

The increased sensitivity of momentum redirection is a potentially important property of the proposed toroidal configuration.

It suggests that geometric deformation can produce a substantial change in the electromagnetic momentum balance.

Whether this change generates an effective restoring force depends on the direction and spatial distribution of the associated stresses.

8.5. Restoring Force and Local Energy Stability

A stable torus requires a restoring response that opposes displacement from equilibrium.

Let F_R denote the generalized radial restoring force.

The equilibrium condition is:

$$F_R(R_0) = 0$$

For local stability:

$$\left.\frac{dF_R}{dR}\right|_{R_0} < 0$$

The corresponding effective restoring stiffness is defined by:

$$K = -\left.\frac{dF_R}{dR}\right|_{R_0}$$

A positive value of K represents a locally restoring radial response.

For a small displacement:

$$y = R - R_0$$

the linear restoring-force approximation becomes:

$$F_R = -Ky$$

An equivalent description can be formulated using the effective deformation energy:

$$U_d = \frac{1}{2}Ky^2$$

The equilibrium is locally stable with respect to this deformation mode when the deformation energy has a local minimum at y=0.

The restoring stiffness is associated with the detailed spatial structure of the electromagnetic fields.

In particular, the strongly nonuniform electric-field geometry proposed in Part A1 determines how local electromagnetic stresses change during compression and expansion.

Because this field geometry has not yet been expressed as a complete spatial function suitable for integration, the numerical value of K remains undetermined.

The present analysis therefore establishes a necessary quantitative stability condition without assigning an unsupported restoring-force coefficient.

8.6. Oscillatory Response to a Small Disturbance

The geometric stress tests demonstrate that electromagnetic energy density and the characteristic momentum-redirection effect respond differently to changes in torus dimensions. However, these scaling relations alone do not determine whether a disturbed torus returns toward equilibrium.

Local stability depends on the direction of the resulting electromagnetic restoring force and on the subsequent dynamical response.

As established in Chapter 6, small radial disturbances can be described by an effective equation of motion:

$$M_{\text{eff}}\ddot{y} + \Gamma\dot{y} + Ky = 0$$

Here, y represents the radial displacement from equilibrium, M_eff is the effective electromagnetic inertia, Γ is the damping coefficient, and K is the restoring stiffness.

For positive effective inertia, the sign of K determines the local restoring tendency.

If K > 0, the electromagnetic response opposes small radial deviations. With positive damping, the resulting motion approaches equilibrium.

For an underdamped response, the condition is:

$$0 < \Gamma < 2\sqrt{KM_{\text{eff}}}$$

In this regime, the torus may undergo transient oscillations whose amplitudes decrease as energy is dissipated.

If K < 0, the effective force increases rather than reduces a small displacement, indicating local instability.

The geometric stress tests provide the deformation dependence of characteristic electromagnetic quantities, but the sign and magnitude of K cannot be obtained from their scaling exponents alone.

Their determination requires the spatial electric- and magnetic-field distributions and the corresponding Maxwell stress response.

The oscillatory model therefore establishes the dynamical conditions for relaxation, while the detailed electromagnetic field geometry must determine whether these conditions are satisfied by the proposed toroidal structure.

8.7. Competition Between Geometric Relaxation and Energy Loss

During the relaxation of an electromagnetic torus, two processes may occur simultaneously. Electromagnetic restoring stresses tend to reduce geometric deviations from equilibrium, while radiative energy loss changes the equilibrium dimensions themselves.

For the assumed equilibrium scaling relation:

$$R_{\text{opt}} \propto W^{1/3}$$

a decrease in retained positive structural energy W causes a reduction in the equilibrium radius.

To describe the relative geometric deformation, introduce:

$$x = \ln\left(\frac{R}{R_{\mathrm{opt}}(W)}\right)$$

For small deviations, x approximately equals the relative displacement from equilibrium. Positive x corresponds to a radius larger than the instantaneous equilibrium radius.

Differentiation gives:

$$\frac{dx}{dt} = \frac{1}{R}\frac{dR}{dt} - \frac{1}{3W}\frac{dW}{dt}$$

As a simplified relaxation model, assume that geometric adjustment reduces the deformation at a rate:

$$\left(\frac{dx}{dt}\right)_{\mathrm{geom}} = -\frac{x}{t_g}$$

Here, t_g represents the characteristic geometric relaxation time.

Assume also that the fractional radiative energy-loss rate increases with the square of the deformation:

$$\frac{1}{W}\frac{dW}{dt} = -\frac{kx^2}{t_c}$$

Here, k is a dimensionless coefficient describing the strength of radiative losses, while t_c is the characteristic circulation time of electromagnetic energy.

Combining the geometric and radiative contributions gives:

$$\frac{dx}{dt} = -\frac{x}{t_g} + \frac{kx^2}{3t_c}$$

The first term reduces geometric deformation through relaxation. The second term describes an increase in relative deformation caused by the inward shift of the equilibrium radius during energy loss.

For positive deformation, x > 0, the two contributions oppose one another. Their magnitudes become equal when:

$$x_{\mathrm{crit}} = \frac{3t_c}{kt_g}$$

For the illustrative values:

$$k = 1, \qquad t_g = 10t_c$$

the threshold is:

$$x_{\mathrm{crit}} = 0.30$$

Below this threshold, geometric relaxation dominates and positive deformation decreases. Above it, the radiative contribution exceeds the assumed relaxation rate, causing the relative deformation to increase within this simplified model.

The threshold represents a change in the direction of deformation evolution, not a physical disintegration limit.

In particular, an increasing relative deformation does not necessarily mean that the actual torus radius is expanding. The equilibrium radius may simply be decreasing more rapidly than the actual radius can adjust.

The model is phenomenological. Neither the relaxation time nor the deformation-dependent radiation coefficient has yet been calculated from the complete electromagnetic field configuration. Consequently, the numerical value of the threshold is illustrative rather than a prediction of a universal stability limit.

The actual structural stability of the torus continues to depend on the electromagnetic restoring stresses, the spatial field geometry, and the ability of the configuration to retain its internal organization.

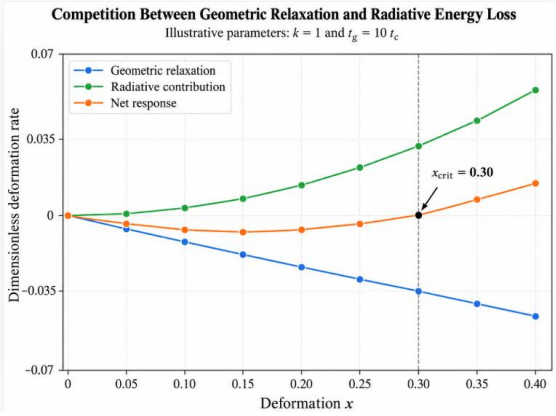


Figure 5 – Competition Between Geometric Relaxation and Radiative Energy Loss. *The diagram compares the geometric relaxation term, the radiative contribution, and their resulting net response as functions of deformation x. For the illustrative parameters k=1 and* $t_g = 10t_c$*, the two effects balance at* $x_{\mathrm{crit}} = 0.30$*, where the net response changes sign. This point represents a conditional threshold within the simplified model, not a calculated physical disintegration limit of the torus.*

8.8. Local Stability Versus Structural Failure

The preceding calculations distinguish two fundamentally different concepts.

Local stability describes the response to small disturbances around an existing equilibrium configuration.

Structural failure involves a loss of the ability to maintain the electromagnetic organization of the torus.

A locally stable torus can return toward equilibrium following small perturbations even if sufficiently large disturbances eventually produce major structural changes.

Conversely, increasing radiative energy loss does not automatically imply immediate disintegration. Radiation may reduce positive expansive energy and improve the relative energetic binding, as described in Chapter 7.

The response to a large disturbance therefore depends on both the geometric restoring mechanism and the ability of the field configuration to retain sufficient coherence.

In the present model, structural instability may arise if the electromagnetic restoring response becomes ineffective, if energy leakage becomes excessive, or if the field geometry changes beyond the range in which the original toroidal configuration remains applicable.

The numerical boundaries of these regimes cannot yet be determined without a detailed spatial model of the electric field and its coupling to the magnetic field.

Large changes in field geometry may also lead to a different bound configuration rather than complete disintegration. Such transformations, including possible supertorus formation, are reserved for Part A3.

8.9. Results of the Local Stress Tests

The analysis establishes three main results.

First, geometrically similar deformation produces predictable changes in average electromagnetic energy density. The inverse-cubic dependence follows directly from the assumed conservation of energy and toroidal volume scaling.

Second, the characteristic local momentum-redirection effect exhibits an inverse-fourth-power dependence on the geometric scale. This greater sensitivity provides a quantitative basis for investigating electromagnetic restoring mechanisms.

Third, a phenomenological relaxation model demonstrates how geometric adjustment and radiative energy loss can jointly influence the evolution of a disturbed torus.

These results support a mathematical framework in which electromagnetic momentum transport, geometric restoring stresses, and radiation participate in the relaxation of a toroidal structure.

The actual local stability of the proposed electromagnetic torus remains dependent on

the sign and magnitude of the net restoring stiffness, which must ultimately be determined from the spatial electric- and magnetic-field configuration.

The following chapter considers the limits of the toroidal stability regime and the possible consequences of larger structural deformations. [1,3,8]

9. Stability Limits and Structural Deformation

9.1. Beyond Local Geometric Stability

The preceding chapters examined the stability of electromagnetic toroids under relatively small geometric disturbances. Within this regime, restoring electromagnetic stresses, transient oscillations, and radiation may enable the structure to approach a stable equilibrium.

At larger deformations, the linear stability model becomes insufficient. The spatial geometry of the electric and magnetic fields may change substantially, and the geometric ratio between the major and minor radii need no longer remain constant.

However, the loss of the original toroidal geometry does not necessarily imply the loss of electromagnetic binding. The structure may remain energetically bound even when its internal field configuration undergoes significant deformation.

9.2. Energetic Binding and Structural Persistence

As established in Chapter 7, the reduction of positive expansive energy can increase the relative dominance of negative binding energy without requiring an increase in its absolute magnitude.

This suggests that a torus may remain structurally persistent even when its electric-field geometry departs significantly from the original configuration.

Two different stability limits must therefore be distinguished: the loss of the original geometric equilibrium and the complete loss of electromagnetic structural binding.

The proposed model allows the possibility that geometric instability occurs before structural disintegration. In such a case, the torus may reorganize into another bound electromagnetic configuration rather than decay completely.

The corresponding transition limits depend on the nonlinear spatial geometry of the electric field and the resulting electromagnetic stresses. Their numerical determination remains beyond the present model.

9.3. From Toroidal Stability to Structural Transformation

Large electric-field deformations may lead to substantial changes in electromagnetic energy-flow paths and the overall organization of the structure.

Two possible outcomes must be considered: transformation into a different bound configuration or loss of electromagnetic confinement.

Of particular interest is the proposed transition from an individual electromagnetic torus toward a more complex supertorus structure.

The present article establishes the energetic and dynamical framework for investigating such transformations but does not attempt to determine their detailed geometry or stability conditions.

These questions form the central subject of Part A3, where large electromagnetic field deformations and supertorus formation are examined. [6,7,8]

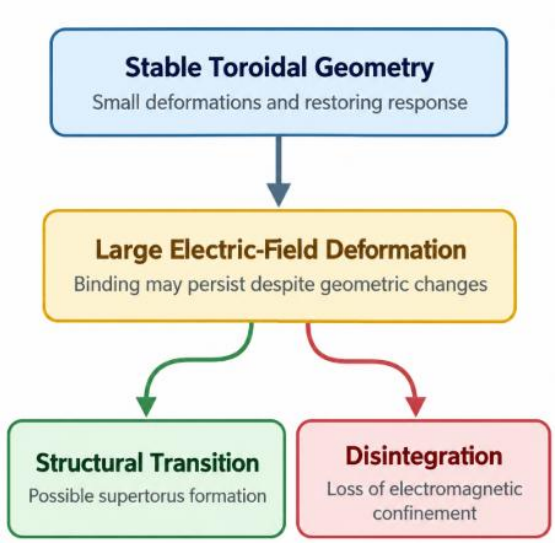


Figure 6 – Stability Limits and Structural Transformation. *Small disturbances may be stabilized through electromagnetic restoring effects and radiative relaxation. Larger electric-field deformations may lead either to structural reorganization or to disintegration. The possible formation of a bound supertorus is investigated in Part A3.*

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10. Quantitative Results and Model Limitations

10.1. Energy and Geometric Scaling

The preceding analysis establishes a quantitative relationship between the energy of an electromagnetic torus and its equilibrium geometry.

The model assumes a constant geometric ratio between the major and minor radii and an approximately constant characteristic energy density in equilibrium.

Under these assumptions, the equilibrium dimensions follow:

$$R(W) = R_0 \left(\frac{W}{W_0}\right)^{1/3}$$

$$a(W) = a_0 \left(\frac{W}{W_0}\right)^{1/3}$$

For two identical toroids merging without radiative losses, the retained energy doubles, and both radii increase by a factor of approximately 1.260.

Radiation reduces the energy retained by the resulting structure and correspondingly decreases its final equilibrium dimensions.

These relationships provide direct quantitative predictions within the adopted geometric and energetic assumptions.

10.2. Sensitivity to Geometric Deformation

The response to proportional geometric deformation can be described using the relative scale s.

At constant total electromagnetic energy, the average energy density follows:

$$\frac{u}{u_0} = s^{-3}$$

If the characteristic curvature radius of the electromagnetic energy-flow paths scales proportionally with the torus dimensions, the local momentum-redirection force density follows the approximate relation:

$$\frac{f_S}{f_{S0}} = s^{-4}$$

The fourth-power dependence indicates greater sensitivity to deformation than the corresponding energy-density change.

For a 5% contraction, the calculated increase in energy density is approximately 16.6%, while the characteristic momentum-redirection effect increases by approximately 22.8%.

For a 5% expansion, the corresponding decreases are approximately 13.6% and 17.7%.

These results quantify the geometric sensitivity of the proposed mechanism. They do not independently establish the magnitude or direction of the net restoring force.

10.3. Local Stability and Oscillatory Relaxation

The local stability of an electromagnetic torus depends on the response of its internal field configuration to small geometric disturbances.

As developed in Chapters 6 and 8, this response can be represented by an effective equation of motion:

$$M_{\text{eff}}\ddot{y} + \Gamma\dot{y} + Ky = 0$$

Here, M_eff represents the effective electromagnetic inertia, K the restoring stiffness, and Γ the damping coefficient.

For positive effective inertia, a locally restoring response requires:

$$K>0$$

When the damping coefficient is positive, disturbances can decrease with time. The oscillatory regime is characterized by:

$$0 < \Gamma < 2\sqrt{KM_{\text{eff}}}$$

Under these conditions, the torus approaches equilibrium through damped oscillations. Stronger damping may produce a non-oscillatory return toward equilibrium.

The geometric stress tests establish the sensitivity of electromagnetic energy density and local momentum redirection to deformation. However, they do not independently determine the sign or magnitude of the resulting restoring stiffness.

The numerical values of M_eff, K, and Γ depend on the spatial electric- and magnetic-field distributions and their evolution during geometric deformation.

Consequently, the present model establishes the mathematical conditions for local relaxation and stability, while their physical realization remains dependent on the detailed electromagnetic field configuration.

10.4. Radiation and Binding Stability

Radiation may contribute to the stabilization of an electromagnetic torus through two complementary processes: damping of geometric oscillations and reduction of positive structural energy relative to negative binding energy.

The proposed model distinguishes positive structural energy W from negative binding energy W_v. Their signed internal energy balance is:

$$U = W + W_v$$

The relative expansive loading is defined as:

$$q = \frac{W}{|W_v|}$$

If radiation reduces positive structural energy while the binding-energy contribution remains approximately constant:

$$W_f = W_i - W_{\text{rad}}$$

$$W_{vf} \approx W_{vi}$$

the relative expansive loading decreases:

$$q_f = \frac{W_i - W_{\text{rad}}}{|W_{vi}|} < q_i$$

Consequently, the existing binding contribution becomes relatively more dominant without requiring an increase in its absolute magnitude.

At the same time, radiation may remove energy associated with transient geometric oscillations, allowing their amplitudes to decrease when an effective restoring mechanism is present.

These effects concern different aspects of stabilization. Oscillation damping describes dynamical relaxation, whereas decreasing q describes a change in the internal energetic balance.

Both processes must be included consistently in the overall energy conservation equation to avoid double counting radiative losses.

The analysis therefore identifies two possible contributions of radiation to toroidal stabilization. However, neither reduced oscillation amplitude nor increased relative binding dominance independently establishes permanent structural stability. This requires a dynamically persistent electromagnetic field configuration with an appropriate restoring response.

10.5. Quantities Requiring Further Determination

Several model parameters remain undetermined numerically.

The most important are the effective restoring stiffness K, the electromagnetic inertia coefficient, the radiative damping coefficient Γ, and the limits of admissible geometric deformation.

Their evaluation requires a sufficiently detailed description of the spatial electric- and magnetic-field configurations.

Of particular importance is the nonlinear spatial geometry of the electric field introduced in Part A1. Its field orientation, gradients, and geometric response determine the distribution of electromagnetic stresses and the direction of the Poynting vector.

Although Maxwell's equations are linear in vacuum, the spatial structure of their solutions and the geometric dependence of the resulting stresses can be highly complex.

The principal mathematical limitation is therefore the absence of a complete spatial field solution for the proposed torus, rather than the absence of conservation laws or governing electromagnetic equations.

A complete evaluation must also establish the physical definition of the negative binding-energy contribution and its relationship to the positive electromagnetic field energy.

These unresolved quantities prevent a numerical prediction of the natural oscillation frequency, the absolute radiation rate, and the critical deformation associated with structural failure.

10.6. Scope of the Quantitative Results

The present analysis establishes a set of quantitative relationships connecting electromagnetic torus merging, equilibrium geometry, momentum transport, and structural stabilization.

Under the assumptions of constant geometric ratio and equilibrium energy density, the characteristic torus dimensions scale with the cube root of positive structural energy. Merging two identical toroids without radiative losses therefore doubles their combined volume and increases both characteristic radii by approximately 26%.

Geometric deformation produces different sensitivities of electromagnetic energy density and local momentum redirection. At constant energy, these quantities vary with the inverse third and fourth powers of the geometric scale, respectively. This difference provides a quantitative basis for investigating electromagnetic restoring mechanisms.

The effective dynamical model establishes the conditions for local stability and damped oscillations. Radiation may additionally reduce geometric excitation and increase the relative dominance of negative binding energy by decreasing positive structural energy.

These results follow from the adopted geometric, energetic, and dynamical assumptions. The absolute restoring stiffness, effective inertia, radiation rate, and structural stability limits remain dependent on the detailed spatial electromagnetic field configuration.

Table 2 summarizes the principal quantitative relationships and distinguishes the derived scaling laws from the stability conditions introduced through the effective dynamical model.

The next stage of the investigation is to determine whether the proposed electromagnetic field geometry produces the restoring stresses and energy confinement required for a persistent toroidal structure. [1,7,8]

Quantity	Result	Basis
Equilibrium radii	$R, a \propto W^{1/3}$	Constant β and energy density
Equal-torus merging	$R_f/R_0 = 2^{1/3}$	Energy conservation, no losses
Energy density	$u/u_0 = s^{-3}$	Constant energy
Momentum-redirection scale	$f_S/f_{S0} = s^{-4}$	Curvature scaling
Local restoring condition	$K > 0$	Effective radial dynamics
Damped oscillation	$0 < \zeta < 1$	Linear oscillator model
Relative expansive loading	$q = E/\lvert E_v \rvert$	Proposed energy partition

Table 2 – Summary of Quantitative Results. *The table summarizes the principal mathematical relationships connecting torus energy, equilibrium geometry, electromagnetic momentum redirection, and structural stability. Geometric scaling and deformation sensitivity follow from the adopted model assumptions, while restoring forces and damped oscillations are described by their stability conditions. The absolute values of the corresponding coefficients remain dependent on the detailed spatial electromagnetic field configuration.*

11. Discussion

The analysis presented in this article connects the merging of electromagnetic toroids with geometric expansion, electromagnetic momentum redistribution, transient oscillations, radiation, and structural stabilization. These processes are treated as successive and mutually related stages in the evolution of a toroidal electromagnetic structure.

The central proposition is that geometrically similar and phase-compatible toroids can form a common electromagnetic configuration. Following merging, the resulting structure contains more energy than either initial torus and must adapt its geometry and internal field distribution to the new energetic conditions.

The adopted equilibrium model provides a simple relationship between energy and geometric dimensions. With an approximately constant ratio between the major and minor radii and a characteristic equilibrium energy density, both radii increase with the cube root of the contained energy. Consequently, doubling the energy increases each radius by approximately 26%, while the occupied volume doubles.

This relationship offers a mechanism for the growth of electromagnetic structures without requiring a permanent increase in the characteristic electric-field amplitude. The additional energy can be accommodated through geometric expansion and redistribution of the electromagnetic fields.

An essential aspect of the proposed process is that the transition toward equilibrium is not instantaneous. Electromagnetic fields carry momentum, and changes in their spatial organization involve momentum transport. The newly formed torus may therefore overshoot its equilibrium dimensions and undergo transient geometric oscillations.

The redirection of the Poynting vector provides a possible physical connection between field

geometry and the electromagnetic stresses involved in this adjustment. Under geometrically similar deformation, the average energy density varies with the inverse third power of the characteristic radius, while the local momentum-redirection force-density scale varies with the inverse fourth power.

This difference demonstrates the strong sensitivity of electromagnetic momentum redistribution to geometric deformation. Even relatively small changes in torus dimensions can produce appreciable changes in the local momentum-transfer scale.

However, the magnitude of this effect alone does not determine the direction of the resulting force. Stable geometric relaxation requires a net restoring response that opposes displacement from equilibrium. Its quantitative determination depends on the detailed electric- and magnetic-field geometry, particularly the orientation and gradients of the spatially nonuniform electric field.

The present model therefore identifies the geometric sensitivity and mathematical conditions required for stabilization, while leaving the absolute restoring-force coefficient to be determined from a more complete electromagnetic field solution.

Radiation introduces a second important aspect of the stabilization process. Deviations from the closed energy-flow configuration may permit part of the electromagnetic energy to escape. Such losses can damp transient oscillations and reduce the energy retained by the structure.

The analysis also distinguishes positive expansive energy from negative binding energy. Within this proposed energy description, radiation can decrease positive energy while the magnitude of the negative binding contribution remains approximately unchanged.

The consequence is a reduction in the relative expansive loading of the binding structure. The absolute binding energy need not increase for its relative dominance to become greater.

This distinction is particularly important. Structural stabilization is not attributed solely to increasing binding energy, but also to reducing the positive energetic contribution that the existing binding mechanism must contain.

Two complementary aspects of stabilization therefore emerge. Electromagnetic restoring stresses can limit geometric deformation, while radiative energy loss can reduce transient excitation and expansive loading. Together, these effects provide a possible route from an initially disturbed electromagnetic configuration toward a more strongly binding-dominated state.

The mathematical description of damped oscillations shows how such a transition can occur when the effective restoring stiffness is positive and radiative damping remains compatible with oscillatory relaxation. The equilibrium dimensions may also change during radiation, because the optimal geometry depends on the energy retained by the torus.

The resulting evolution need not follow a simple monotonic expansion or contraction. Instead, the structure may oscillate around a gradually changing equilibrium configuration while progressively losing energy.

The analysis further indicates that geometric stability and structural persistence should not be regarded as identical properties. A torus may become unstable with respect to its original geometry without necessarily losing its electromagnetic binding. Substantial deformation of the electric-field structure may therefore lead to reorganization into another bound configuration rather than complete disintegration.

This possibility establishes a connection with Part A3, where large electromagnetic field

deformations and the proposed formation of supertoroidal structures are investigated.

Several aspects of the present description remain dependent on the underlying electromagnetic field model. In particular, the merging process requires a mechanism capable of producing a persistent combined configuration, while local stability requires a net restoring response. These properties cannot be established from field superposition or geometric scaling alone.

The negative binding-energy contribution also requires a consistent physical definition and a complete conservation balance. In conventional vacuum electrodynamics, electric and magnetic field-energy densities are nonnegative, and permanent self-confinement does not follow simply from the curvature of energy-flow paths. The proposed binding mechanism must therefore be demonstrated through a sufficiently complete field configuration or an explicitly specified extension of the model.

The principal limitation of the quantitative analysis is the incomplete mathematical description of the spatial electric-field geometry introduced in Part A1. Its nonuniform shape and changing orientation influence the Poynting vector, the Maxwell stress tensor, and the effective coefficients governing inertia, restoring forces, and radiation.

Despite these remaining requirements, the present analysis establishes a connected mathematical framework relating electromagnetic energy, toroidal geometry, momentum transport, transient relaxation, and relative binding stability.

The resulting picture is one in which the growth and stabilization of electromagnetic structures are not independent phenomena. Merging changes the energy content, energy redistribution changes the geometry, electromagnetic momentum transport governs the transient response, and radiation can influence both geometric relaxation and energetic binding.

This framework provides a basis for further quantitative investigation of electromagnetic toroidal structures and their possible development into more complex configurations.

12. Conclusions

This article develops a quantitative framework for the merging, geometric growth, and stabilization of electromagnetic toroidal structures, extending the model introduced in Part A1.

The proposed mechanism considers the merging of geometrically similar and phase-compatible toroids into a unified higher-energy configuration. Under the assumptions of constant geometric ratio and equilibrium energy density, both characteristic radii scale with the cube root of positive structural energy. Merging two identical toroids without radiation and without a change in their combined binding-energy contribution therefore doubles their positive energy and increases their equilibrium radii by approximately 26%.

The analysis identifies electromagnetic momentum redirection as a possible mechanism contributing to geometric stabilization. Under geometrically similar deformation at constant energy, the average energy density varies with the inverse third power of the characteristic radius, while the local momentum-redirection force-density scale varies with the inverse fourth power. The resulting sensitivity provides a quantitative basis for investigating electromagnetic restoring stresses.

Electromagnetic inertia may produce transient oscillations during geometric relaxation. Radiation can contribute to their damping while simultaneously reducing the energy

retained by the structure and shifting its equilibrium dimensions.

A further result concerns the relationship between positive structural energy W and negative binding energy W_v. Radiative reduction of W can increase the relative dominance of the binding contribution without increasing its absolute magnitude. The model therefore identifies two complementary effects of radiation: damping geometric disturbances and reducing relative expansive loading.

Energetic binding dominance, local geometric stability, and long-term structural persistence are distinct properties. The derived scaling relations and effective stability conditions establish a mathematical framework, while demonstrating persistent electromagnetic confinement requires a complete spatial field solution and a physically consistent binding-energy model.

The central contribution of Part A2 is to connect structural merging, geometric expansion, electromagnetic momentum transport, and radiative relaxation within a common theoretical description. It provides a foundation for investigating larger electromagnetic structures and the possible formation of supertoroids in Part A3.

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