

Unified Electromagnetic Theory of Matter and Light

Section A — Structure of Matter

Part II - The Electron as an Electromagnetic Supertorus

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Abstract

*This paper develops a geometrical and electromagnetic model of the electron based on a **supertorus**, a toroidal structure characterized by two coupled circulations and two characteristic radii. The objective is to investigate whether several experimentally known properties of the electron, normally introduced as independent intrinsic quantities, can be related to a common underlying structure.*

The analysis connects the supertorus geometry with the electron charge, rest energy, angular momentum, magnetic moment, internal electric and magnetic fields, displacement current, and circulating charge. Particular attention is given to the two characteristic radii and to the inner–outer asymmetry inherent in toroidal geometry. The resulting equations form a system of mutually connected constraints in which geometry determines possible charge and field configurations, while the measured properties of the electron restrict the allowed geometry.

The electron rest energy provides the energy constraint

$$U = m_e c^2,$$

while its angular momentum requires

$$L = \frac{\hbar}{2}.$$

Charge circulation generates current and magnetic moment,

$$I = \frac{q}{T}, \quad \mu = I A,$$

thereby connecting electromagnetic circulation directly with geometry. The model consequently seeks a single configuration satisfying

$$q = -e, \quad U = m_e c^2, \quad L = \frac{\hbar}{2}, \quad \mu = \mu_e.$$

The principal result of the analysis is that these conditions need not be treated as completely independent requirements. Within the supertorus they become parts of an interconnected geometrical and electromagnetic system. The model therefore provides a quantitative framework for investigating whether the measured properties of the electron can emerge as different manifestations of one underlying electromagnetic structure.

The supertorus does not merely propose a shape for the electron. It defines a program in which progressively fewer electron properties are introduced as independent inputs and progressively more are sought as consequences of geometry and electromagnetic dynamics. The results presented here indicate a concrete direction toward a structural description of the electron in which charge, mass-energy, angular momentum, and magnetic moment may ultimately acquire a common physical origin.

1. Introduction

The electron is one of the simplest known elementary particles in terms of its experimentally observed properties. It is characterized by a precisely defined rest mass, electric charge, spin, and magnetic moment. At the same time, the physical origin of these properties remains outside the scope of a classical structural description. In the Standard Model, the electron is treated as an elementary particle without a resolved internal structure, and its properties are introduced through the corresponding quantum fields and parameters.

This article investigates a different question. Rather than assuming a particular microscopic structure and subsequently calculating its properties, we ask whether the known properties of the electron can be used as boundary conditions for reconstructing a possible electromagnetic structure. The approach is therefore primarily inverse: the experimentally known electron serves as the boundary condition, while the geometry and internal dynamics of the proposed structure are quantities to be determined.

The starting point is a closed electromagnetic configuration referred to here as an **electromagnetic supertorus**. It is assumed to consist of a sequence of mutually coherent toroidal electromagnetic cells. The individual cells oscillate synchronously, but with a systematic phase shift between neighboring cells. After one complete circuit around the supertorus, the accumulated phase shift is 2π .

The electromagnetic configuration is therefore closed both geometrically and in phase.

This phase closure has an important consequence. The internal electromagnetic motion cannot in general be represented by a single current circulating along a circle of radius R . The inner and outer parts of the supertorus follow paths of different lengths. If they remain phase synchronized, their local phase velocities must consequently differ. The electromagnetic motion is therefore intrinsically distributed across the toroidal cross-section.

A second requirement concerns radiation. A stationary electron does not continuously lose its rest energy through electromagnetic radiation. Any electromagnetic model of the electron must therefore admit an internally dynamic configuration without a net outward radiative energy flux. In the present approach, this condition is considered primarily as a problem of coherent superposition: the fields of the individual toroidal cells must combine so that the outgoing radiative components cancel, while a non-radiative external electric field may remain. A complete Maxwell solution of this boundary-value problem is not assumed in advance and remains an important test of the model.

The known properties of the electron provide several unusually restrictive constraints. Its total rest energy is

$$U = m_e c^2$$

its electric charge is

$$Q = -e$$

its intrinsic angular momentum is

$$J = \frac{\hbar}{2}$$

and its magnetic moment is close to the Bohr magneton,

$$\mu_B = \frac{e\hbar}{2m_e}$$

with a small experimentally known anomalous contribution. A viable electromagnetic structure must reproduce these quantities simultaneously rather than explain each of them by an unrelated assumption.

The central objective of this work is therefore not merely to demonstrate that a toroidal electromagnetic configuration can be imagined. The stronger question is whether a **single geometrical and dynamical model** can satisfy several independent electron constraints at the same time.

Two characteristic dimensions are introduced: the major radius R of the supertorus and the minor radius a of its cross-section. Phase closure, together with the characteristic electromagnetic propagation condition, relates the angular frequency to the major radius. When the electron rest energy and the relation $U = \hbar\omega$ are used as boundary conditions, a characteristic major radius of the order

$$R \sim \frac{\hbar}{m_e c}$$

emerges, corresponding to the reduced Compton scale of the electron.

The angular-momentum condition provides an additional geometrical constraint. Within the simplest symmetric approximation developed below, it leads to a minor radius of approximately

$$a \sim \frac{R}{4}$$

The resulting geometry then permits estimates of the internal electric and magnetic field

amplitudes and, independently, an examination of the magnetic moment associated with the circulating charge asymmetry.

An important distinction is made throughout the analysis between **displacement current, electromagnetic energy flow, and effective circulation of net charge**. A sinusoidally oscillating displacement current has zero time average and therefore cannot simply be identified with the persistent magnetic moment of the electron. By contrast, the Poynting vector may possess a nonzero time-averaged circulation, while a phase-locked asymmetric charge distribution can generate a persistent magnetic moment. Keeping these three phenomena separate removes ambiguities that arise when the supertorus is approximated as a conventional current loop.

The inner and outer regions of the supertorus also cannot be treated as equivalent. Their path lengths, local phase velocities, available volumes, field densities, and contributions to the magnetic moment differ. This geometrical asymmetry becomes particularly important when the measured electron magnetic moment is imposed as a constraint on the spatial distribution of charge.

The purpose of the following analysis is thus to determine how far the supertorus hypothesis can be developed quantitatively using a minimal set of assumptions. Experimental electron properties are used explicitly as boundary conditions where necessary; quantities that follow from these conditions are distinguished from quantities that remain assumptions of the model. This distinction is essential because numerical agreement obtained by inserting a measured property cannot subsequently be regarded as an independent prediction.

The proposed supertorus should therefore be understood as a **testable electromagnetic model rather than an established description of the electron**. Its relevance depends on whether the same internal geometry can

simultaneously account for rest energy, angular momentum, electric charge, and magnetic moment, and ultimately whether a self-consistent non-radiating electromagnetic field configuration satisfying the required boundary conditions can be demonstrated. The calculations developed in the following sections provide the first quantitative tests of these requirements.

2. Geometry of the Electromagnetic Supertorus

The electromagnetic supertorus is assumed to be formed from a sequence of mutually coherent toroidal electromagnetic cells. Each cell represents a locally closed electromagnetic structure, while the cells themselves are arranged along a larger circular path. When the sequence closes upon itself, a second, larger toroidal structure is formed. We refer to this closed structure as the **electromagnetic supertorus**.

Two geometrical scales are required to describe the structure. The major radius R specifies the distance from the central axis of the supertorus to the center of its cross-section, while the minor radius a specifies the characteristic radius of that cross-section. At this stage, both R and a are independent parameters; their values will be constrained later by the measured properties of the electron.

The volume of an ideal torus with these dimensions is

$$V = 2\pi^2 R a^2$$

This expression will later be used to relate the total electromagnetic energy to the characteristic electric and magnetic field amplitudes inside the supertorus.

The individual toroidal cells are not assumed to oscillate independently. They oscillate at the same frequency and are mutually phase locked. However, neighboring cells have a small

systematic phase difference. If the supertorus contains N cells and the total phase change around the complete structure is 2π , the phase difference between neighboring cells is

$$\Delta\phi = \frac{2\pi}{N}$$

The phase of the n -th cell may therefore be written as

$$\phi_n = \phi_0 + \frac{2\pi n}{N}$$

After one complete circuit,

$$\phi_N - \phi_0 = 2\pi$$

Thus, the first cell and the last cell meet in the same physical phase, while the phase has completed one full cycle around the supertorus. This represents the simplest closed mode, with winding number one. Higher-order modes with several complete phase cycles are in principle possible, but only the fundamental mode is considered here.

In the continuous limit, the phase along the major circumference may be represented by

$$\phi(s, t) = ks - \omega t$$

where s is the coordinate along the major circumference, k is the corresponding wave number, and ω is the angular frequency.

For the fundamental closed mode, the phase must increase by 2π over the circumference $2\pi R$. Therefore,

$$k(2\pi R) = 2\pi$$

and hence

$$k = \frac{1}{R}$$

The corresponding wavelength of the phase pattern along the major circumference is consequently

$$\lambda = 2\pi R$$

This phase arrangement has an important geometrical consequence. The electromagnetic motion inside the supertorus

cannot be represented as motion along a single circle of radius R . Each elementary toroidal cell contains an internal circulation, while the progressive phase shift between neighboring cells advances this circulation around the major circumference. The resulting electromagnetic motion therefore has a helical character.

A particularly important distinction arises between the inner and outer regions of the supertorus. Their characteristic distances from the central axis are

$$r_{\text{in}} = R - a$$

and

$$r_{\text{out}} = R + a$$

Consequently, the corresponding circumferences are different:

$$L_{\text{in}} = 2\pi(R - a)$$

$$L_{\text{out}} = 2\pi(R + a)$$

Nevertheless, the inner and outer parts belong to the same coherent electromagnetic oscillation. They must therefore complete the corresponding phase evolution within the same period T . This requires a common angular phase velocity

$$\omega = \frac{2\pi}{T}$$

while their local linear phase velocities are different:

$$v_{\text{in}} = \omega(R - a)$$

$$v_{\text{out}} = \omega(R + a)$$

Thus,

$$\frac{v_{\text{out}}}{v_{\text{in}}}$$

=

$$\frac{R + a}{R - a}$$

The difference between these velocities is not introduced as an independent assumption. It

follows directly from the requirement that paths of different lengths remain phase synchronized.

If the characteristic phase velocity at the central radius R is identified with c ,

$$\omega R = c$$

then the local phase velocity across the toroidal cross-section takes the simple form

$$v(r) = \omega r = c \frac{r}{R}$$

Accordingly,

$$v_{\text{in}} = c \left(1 - \frac{a}{R}\right)$$

$$v_{\text{out}} = c \left(1 + \frac{a}{R}\right)$$

These quantities should presently be interpreted as velocities of phase progression within the coherent structure. In particular, $v_{\text{out}} > c$ would not by itself establish superluminal transport of energy or information. The distinction between phase progression and electromagnetic energy transport will become important when the Poynting flow is considered.

The different inner and outer circumferences also imply that equal total displacement-current amplitudes need not correspond to equal current densities. For an individual toroidal cell, symmetry permits the amplitudes of the total inner and outer displacement currents to be equal,

$$I_{\text{in},0} = I_{\text{out},0}$$

while their local current densities may differ because the corresponding electromagnetic fields occupy different effective volumes and cross-sectional areas:

$$J_{\text{in}} \neq J_{\text{out}}$$

This distinction becomes essential when the supertorus is treated as a three-dimensional structure rather than as a thin current loop.

The geometry therefore contains two simultaneous forms of closure. The first is

spatial closure: the chain of elementary toroidal cells closes into a torus of major radius R . The second is **phase closure:** the electromagnetic phase advances by exactly 2π during one circuit. Together they produce a coherent structure in which local oscillation, circulation within individual cells, and global phase progression around the supertorus are coupled.

At this stage, the geometry does not determine the absolute values of R and a . It establishes only the relationships required for coherent closure. The experimentally known rest energy and angular momentum of the electron will provide additional boundary conditions from which these dimensions can subsequently be estimated.

3. Phase Closure and Synchronous Internal Motion

The geometrical closure of the supertorus alone is not sufficient to define a stable electromagnetic structure. The electromagnetic oscillation must also close in phase. The individual toroidal cells are therefore assumed to oscillate at the same angular frequency ω , while a constant phase increment exists between neighboring cells. In the fundamental mode, the accumulated phase change after one complete circuit is exactly 2π .

For a continuous description along the major circumference, the phase may be written as

$$\phi(s, t) = ks - \omega t$$

The phase-closure condition is

$$k(2\pi R) = 2$$

which gives

$$k = \frac{1}{R}$$

The corresponding phase wavelength along the major circumference is therefore

$$\lambda_\phi = 2\pi R$$

This condition describes a single complete phase cycle distributed around the supertorus. The electromagnetic state at the point where the sequence closes is therefore identical in phase to the state at its beginning.

3.1 Synchronous Oscillation of the Toroidal Cells

Phase closure does not mean that all cells have the same instantaneous phase. Rather, all cells oscillate at the same frequency while their phases are systematically shifted according to their positions.

For N cells,

$$\Delta\phi = \frac{2\pi}{N}$$

and the phase of the n -th cell may be written as

$$\phi_n(t) = \omega t + \phi_0 + \frac{2\pi n}{N}$$

Every cell therefore completes one oscillation during the same period

$$T = \frac{2\pi}{\omega}$$

while the phase pattern itself progresses continuously around the supertorus.

This distinction is essential. A given cell performs a local electromagnetic oscillation, whereas the collective phase pattern represents a global motion around the closed structure. The supertorus therefore combines **local oscillation with global phase progression**.

3.2 Inner and Outer Paths

Because the supertorus has a finite minor radius a , its inner and outer regions have different path lengths. Their characteristic radii are

$$r_{\text{in}} = R - a,$$

$$r_{\text{out}} = R + a$$

The corresponding path lengths for one complete revolution are

$$L_{\text{in}} = 2\pi(R - a)$$

$$L_{\text{out}} = 2\pi(R + a)$$

The two paths have different lengths, but they belong to the same coherent oscillation and must complete one phase cycle in the same period T .

Consequently,

$$T_{\text{in}} = T_{\text{out}} = T$$

The corresponding local phase velocities are therefore

$$v_{\text{in}} = \frac{2\pi(R-a)}{T}$$

$$v_{\text{out}} = \frac{2\pi(R+a)}{T}$$

Using

$$\omega = T2\pi,$$

$$v_{\text{in}} = \omega(R - a)$$

$$v_{\text{out}} = \omega(R + a)$$

Thus,

$$v_{\text{out}} > v_{\text{in}}$$

Their ratio is determined entirely by geometry:

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{R+a}{R-a}$$

The velocity difference is therefore not an additional parameter of the model. It is required by synchronous phase evolution along paths of different lengths.

3.3 Characteristic Velocity at the Major Radius

For an electromagnetic structure, it is natural to introduce c as the characteristic propagation velocity at the central radius R . We therefore impose the working condition

$$v(R) = \omega R = c$$

It follows immediately that

$$\omega = \frac{c}{R}$$

This result is also obtained by combining the phase-closure condition

$$k = R1$$

with the local electromagnetic dispersion relation

$$\omega = ck$$

Thus, the geometrical phase closure and the characteristic electromagnetic propagation condition lead independently to the same relation,

$$\omega R = c$$

This relation will become one of the central constraints of the model.

3.4 Velocity Distribution Across the Supertorus

The common angular frequency implies that the local phase velocity varies linearly with the distance r from the central axis:

$$v(r) = \omega r$$

Using $\omega R = c$,

$$v(r) = c \frac{r}{R}$$

The inner and outer velocities therefore become

$$v_{\text{in}} = c \left(1 - \frac{a}{R}\right)$$

$$v_{\text{out}} = c \left(1 + \frac{a}{R}\right)$$

An important consequence is

$$\frac{v_{\text{in}} + v_{\text{out}}}{2} = c$$

Thus, c occupies a natural geometrical position as the phase velocity associated with the central circumference of the supertorus.

At this stage, the quantities v_{in} and v_{out} are interpreted as **local phase velocities of the coherent electromagnetic pattern**, not automatically as velocities of energy, charge, or information. This distinction is essential because the outer phase velocity can exceed c

whenever $a > 0$. No conclusion about superluminal energy transport follows from this phase relation alone.

3.5 Helical Character of the Internal Motion

The progressive phase shift between neighboring toroidal cells also changes the geometrical interpretation of the internal electromagnetic motion.

If all cells had identical phase, their local displacement currents would oscillate without a systematic progression around the major circumference. With a total phase shift of 2π , however, the position of a given phase continuously moves from one cell to the next.

The internal electromagnetic pattern therefore follows a helical path: during one complete progression around the major circumference, the local phase also completes one full cycle associated with the toroidal cells.

This may be represented schematically by two angular coordinates. Let ϕ denote the position around the major circumference and θ the local phase angle associated with the individual toroidal cell. For the fundamental mode,

$$\theta = \phi + \theta_0$$

After one complete circuit,

$$\Delta\phi = 2\pi,$$

$$\Delta\theta = 2\pi$$

The simplest closed internal trajectory is therefore a one-to-one winding of the two angular coordinates.

This helical structure is important because the displacement current does not simply circulate along the central radius R . Its inner and outer portions follow different geometrical paths while remaining part of one coherent electromagnetic cycle.

3.6 Equal Current Amplitudes and Different Current Densities

Consider one toroidal cell separately. The inner and outer portions belong to the same closed

displacement-current oscillation. In the simplest symmetric approximation, conservation of the closed current implies equal amplitudes:

$$I_{in,0} = I_{out,0} = I_0$$

This does not require equal local displacement-current densities. Since

$$I_D = \int_A J_D \cdot dA$$

different effective cross-sectional areas or different volumetric packing of the toroidal cells can give

$$J_{in} \neq J_{out}$$

while maintaining

$$I_{in} = I_{out}$$

This distinction becomes especially important when the inner and outer parts are later evaluated separately in calculations of electromagnetic energy flow, angular momentum, and charge asymmetry.

The phase-closure condition therefore provides considerably more than a periodic boundary condition. It couples the major radius R , the oscillation frequency ω , the inner and outer phase velocities, and the three-dimensional circulation of the electromagnetic field. These relationships form the dynamical framework on which the subsequent energy and angular-momentum constraints are imposed

4. Non-Radiating Character of the Closed Structure

A central requirement of the supertorus hypothesis is that the internal electromagnetic motion does not continuously radiate energy away from the structure. A stationary electron possesses a stable rest energy, and therefore any electromagnetic model of the electron must contain a mechanism by which internal time-dependent fields can remain confined.

The present model does not assume that the electromagnetic fields are static. On the contrary, the individual toroidal cells oscillate and exchange electric and magnetic energy. The required condition is weaker and more specific: the **net outward radiative field must vanish**, even though local electromagnetic oscillation remains present.

This distinction allows the problem to be formulated as one of coherent superposition.

4.1 Superposition of the Fields of the Toroidal Cells

Let the electric field generated by the n-th toroidal cell at an observation point $\mathbf{r}$ be written in complex notation as

$$\mathbf{E}_n(\mathbf{r}, t) = \Re \left[\mathbf{E}_n(\mathbf{r}) e^{i(\omega t + \phi_n)} \right]$$

The total electric field is the superposition of the fields of all N cells,

$$\mathbf{E}_{\text{tot}}(\mathbf{r}, t) = \Re \left[e^{i\omega t} \sum_{n=1}^N \mathbf{E}_n(\mathbf{r}) e^{i\phi_n} \right]$$

For the fundamental closed mode,

$$\phi_n = \phi_0 + \frac{2\pi n}{N}$$

If the radiative contributions of the individual cells have approximately equal amplitudes in the far field, the phase-dependent part of the sum contains the expression

$$\sum_{n=0}^{N-1} e^{i2\pi n/N}$$

This is the sum of the N-th roots of unity, for which

$$\sum_{n=0}^{N-1} e^{i2\pi n/N} = 0$$

Thus, in the ideal symmetric limit, a complete 2π phase distribution around the supertorus can cause the coherent radiative amplitudes to cancel.

This is the simplest mathematical indication of how a time-dependent internal electromagnetic structure may avoid producing a net far-field radiation amplitude.

4.2 Continuous Limit

For a large number of toroidal cells, the discrete sum may be replaced by a continuous phase distribution. If the phase around the major circumference is

$$\phi(\varphi) = \varphi + \phi_0$$

then the corresponding phase integral over one complete revolution is

$$\int_0^{2\pi} e^{i\phi(\varphi)} d\varphi$$

For the fundamental mode,

$$\int_0^{2\pi} e^{i\varphi} d\varphi = 0$$

The cancellation is therefore not dependent on the number of cells. It follows from a complete and uniform distribution of phase over 2π .

This result should not be interpreted as a proof that every component of the electromagnetic field vanishes outside the supertorus. It shows only that a coherent closed phase distribution has the capacity to suppress the leading radiative contribution through destructive interference.

4.3 Radiation and Static External Fields Must Be Distinguished

The absence of radiation does not require

$$\mathbf{E}_{\text{out}} = 0$$

or

$$\mathbf{B}_{\text{out}} = 0$$

everywhere outside the supertorus.

What is required is that the part of the field responsible for a net outward transport of electromagnetic energy vanishes sufficiently far from the structure.

The time-averaged electromagnetic energy flux is described by the Poynting vector,

$$\langle \mathbf{S} \rangle = \left\langle \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \right\rangle.$$

For a non-radiating stationary configuration, the total outward power through a sufficiently large closed surface must satisfy

$$P_{\text{rad}} = \oint \langle \mathbf{S} \rangle \cdot d\mathbf{A} = 0.$$

This is the physically relevant confinement condition.

A nonzero external static electric field can nevertheless remain. In the electron interpretation, the asymptotic electric field must approach the Coulomb form,

$$E(r) \rightarrow \frac{e}{4\pi\epsilon_0 r^2} \quad (r \gg R)$$

Thus, the model separates two fundamentally different external components:

$$\text{radiative field} \rightarrow 0$$

While

$$\text{static electric field} \neq 0.$$

This distinction is essential. The first condition is required for stability; the second is required for the observed electric charge.

4.4 Why a Closed Phase Structure Can Suppress Radiation

A conventional oscillating electric dipole radiates because its time-dependent multipole moment produces a coherent far-field component. In the supertorus, however, the source is distributed around a closed geometry with a continuously varying phase.

At any given instant, a region of the supertorus that contributes a positive radiative amplitude is accompanied by other regions whose phase is shifted and whose far-field contribution can oppose it.

For two components separated by a phase difference of π ,

$$E_2 = -E_1$$

For the full supertorus, the cancellation generalizes continuously over all phases between 0 and 2π .

The physical picture is therefore not that individual toroidal cells fail to generate time-dependent fields. Rather, the complete closed structure may produce no net outgoing radiation because the individual radiative amplitudes cancel in the far field.

4.5 Multipole Interpretation

The superposition argument may also be viewed in terms of electromagnetic multipoles. Far-field radiation is determined not by the existence of local oscillating fields alone, but by the time dependence of the corresponding global multipole moments.

A completely symmetric phase distribution may strongly suppress the lowest-order radiative multipoles. In particular, the net oscillating electric dipole moment may satisfy

$$\mathbf{p}(t) = \int \mathbf{r} \rho(\mathbf{r}, t) dV \approx 0.$$

Similarly, cancellation may occur for other low-order radiative components.

A rigorous demonstration would require evaluating the complete multipole expansion or solving the Maxwell equations with the full toroidal boundary conditions. The present argument therefore establishes a **necessary and plausible cancellation mechanism**, but not yet a complete proof of exact non-radiation.

4.6 Internal Energy Circulation

Suppression of outward radiation does not imply suppression of internal energy flow. Inside the supertorus, the Poynting vector may circulate continuously.

For local electromagnetic fields,

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}.$$

The phase-locked toroidal geometry can therefore support a closed energy-flow pattern in which energy circulates within the structure rather than escaping to infinity.

The qualitative distinction is

$$\oint_{\text{internal}} \mathbf{S} \cdot d\mathbf{l} \neq 0$$

while

$$\oint_{\text{external}} \langle \mathbf{S} \rangle \cdot d\mathbf{A} = 0.$$

The first relation represents internal circulation; the second represents the absence of net radiative loss.

4.7 Stability Condition of the Supertorus

The minimum radiation condition of the model may therefore be summarized as

$$\lim_{r \rightarrow \infty} \oint_{A(r)} \langle \mathbf{S} \rangle \cdot d\mathbf{A} = 0.$$

This condition is weaker than requiring the external electromagnetic field itself to vanish and is compatible with the existence of the static Coulomb field associated with the electron charge.

The phase closure discussed in the preceding section provides a natural mechanism by which this condition may be approached. A complete 2π phase distribution causes the simplest coherent radiative amplitude to cancel exactly in the ideal symmetric approximation.

The remaining question is whether the same cancellation persists when the full three-dimensional field distribution, the finite minor radius, the inner–outer asymmetry, and higher electromagnetic multipoles are included. This remains one of the principal theoretical tests of the supertorus model.

For the quantitative development that follows, the structure will therefore be treated under the explicit working hypothesis that the closed phase configuration suppresses net radiation while allowing internal electromagnetic energy circulation and a non-radiative external field associated with the net electric charge.

5. Electron Rest Energy as a Boundary Condition

The geometry and phase-closure conditions developed above determine relationships between the dimensions and internal frequency of the supertorus, but they do not yet determine its absolute scale. In particular, the phase-closure condition gives

$$\omega R = c$$

but any simultaneous change of R and ω that preserves this product would satisfy the same geometrical condition. An additional physical boundary condition is therefore required.

For a model intended to represent an electron at rest, the most direct such condition is its measured rest energy,

$$U = m_e c^2$$

The central assumption of the present model is that this energy is stored in the internal electromagnetic field of the supertorus. Thus,

$$U_{\text{EM}} = m_e c^2$$

This condition does not yet specify how the energy is distributed between the electric and magnetic fields. It requires only that the integral of the total electromagnetic energy density over the volume of the structure equals the measured rest energy of the electron.

In general,

$$U_{\text{EM}} = \int_V \left(\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right) dV$$

This is the first energetic boundary condition of the model.

5.1 Energy and Internal Oscillation Frequency

The rest-energy condition alone does not yet connect the absolute size R with the oscillation frequency ω . To establish this connection, a second experimentally motivated boundary condition is introduced:

$$U = \hbar\omega$$

Combined with the electron rest energy,

$$m_e c^2 = \hbar\omega$$

The characteristic angular frequency of the internal electromagnetic oscillation is therefore

$$\omega = \frac{m_e c^2}{\hbar}$$

Numerically,

$$\omega \approx 7.76 \times 10^{20} \text{ rad s}^{-1}$$

The corresponding ordinary frequency is

$$f = \frac{\omega}{2\pi}$$

which gives approximately

$$f \approx 1.24 \times 10^{20} \text{ Hz}$$

This frequency is not obtained from the geometry alone. It enters because the experimentally known electron rest energy is connected to the internal frequency through the relation $U = \hbar\omega$.

5.2 Determination of the Major Radius

The preceding section established the phase-closure relation

$$\omega R = c$$

Substituting

$$\omega = \frac{m_e c^2}{\hbar}$$

gives

$$\frac{m_e c^2}{\hbar} R = c$$

Solving for the major radius gives

$$R = \frac{\hbar}{m_e c}$$

The quantity on the right-hand side is the reduced Compton wavelength of the electron,

$$\bar{\lambda}_C = \frac{\hbar}{m_e c}$$

and therefore

$$R = \bar{\lambda}_C$$

Numerically,

$$R \approx 3.86 \times 10^{-13} \text{ m}$$

The major radius of the simplest supertorus model therefore emerges at the reduced Compton scale of the electron.

5.3 Phase Wavelength Around the Supertorus

From the phase-closure condition derived previously,

$$\lambda_\phi = 2\pi R$$

Since

$$R = \frac{\hbar}{m_e c},$$

it follows that

$$\lambda_\phi = 2\pi \frac{\hbar}{m_e c}.$$

Because

$$h = 2\pi\hbar,$$

we obtain

$$\lambda_\phi = \frac{h}{m_e c}$$

The right-hand side is the ordinary Compton wavelength,

$$\lambda_C = \frac{h}{m_e c}$$

and consequently

$$\lambda_\phi = \lambda_C$$

Numerically,

$$\lambda_C \approx 2.43 \times 10^{-12} \text{ m}$$

Thus, one complete phase cycle around the major circumference of the supertorus corresponds to the Compton wavelength.

This correspondence follows from the three conditions

$$U = m_e c^2, \quad U = \hbar \omega, \quad \omega R = c$$

These relations imply

$$2\pi R = \lambda_C$$

5.4 Interpretation of the Result

The appearance of the Compton scale is an important internal consistency result, but its interpretation requires care.

The relation

$$R = \frac{\hbar}{m_e c}$$

has **not** been obtained from classical electromagnetic geometry alone. The quantum relation

$$U = \hbar \omega$$

has been introduced as a boundary condition. Consequently, the emergence of the reduced Compton wavelength cannot at this stage be regarded as an independent prediction of the supertorus model.

Rather, the calculation answers an inverse question:

If an electromagnetic supertorus has the measured rest energy of the electron, satisfies the energy-frequency relation $U = \hbar \omega$, and contains one complete phase cycle around its major circumference, what major radius is required?

The result is

$$R = \overline{\lambda_C}$$

This establishes the absolute length scale on which the remaining properties of the model can be tested.

The distinction between input and prediction is important. From this point onward, R is no longer treated as an adjustable geometrical parameter. Once the electron rest energy, the energy-frequency relation, and the fundamental phase-closure condition have been imposed, the major radius is fixed.

5.5 Rest Energy as Electromagnetic Field Energy

The result allows the total energy condition to be expressed more concretely. The electromagnetic energy stored in the supertorus must satisfy

$$\int_V u_{\text{EM}} dV = m_e c^2$$

where the electromagnetic energy density is

$$u_{\text{EM}} = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$$

Combining the two expressions gives the complete energy condition

$$\int_V \left(\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right) dV = m_e c^2$$

At this stage, however, the minor radius a remains unknown. Consequently, although the ideal toroidal volume is

$$V = 2\pi^2 R a^2$$

its numerical value cannot yet be determined.

The rest-energy boundary condition therefore fixes the characteristic major scale R , but does not by itself determine the complete geometry or the internal field amplitudes.

A second independent constraint is required to determine a . The natural candidate is the intrinsic angular momentum of the electron,

$$J = \frac{\hbar}{2}$$

The next section therefore examines the angular momentum carried by the circulating electromagnetic energy of the supertorus and asks whether the measured electron spin can determine the remaining geometrical scale a .

6. Angular Momentum and the Minor Radius

The rest-energy boundary condition fixed the major radius of the supertorus at the reduced Compton scale,

$$R = \frac{\hbar}{m_e c}$$

The minor radius a , however, remains undetermined. A second independent property of the electron is therefore required. In the present model, the natural constraint is the intrinsic angular momentum of the electron,

$$J = \frac{\hbar}{2}$$

The purpose of this section is to examine whether the angular momentum associated with the circulating electromagnetic energy of the supertorus can determine a .

6.1 Angular Momentum of Circulating Electromagnetic Energy

Electromagnetic fields carry both energy and momentum. For a locally propagating electromagnetic energy distribution with energy U_i , the associated effective momentum may be written, in the present simplified model, as

$$p_i = \frac{U_i}{c^2} v_i$$

where v_i denotes the local velocity associated with the circulating energy flow.

For energy circulating at a characteristic radius r_i , the corresponding angular momentum is

$$J_i = r_i p_i$$

and therefore

$$J_i = \frac{U_i}{c^2} r_i v_i$$

This expression is used here as a reduced description of the angular momentum of the distributed electromagnetic energy. A complete field calculation would instead evaluate the angular-momentum density directly from the Poynting vector.

The corresponding field expression is

$$\mathbf{J}_{\text{EM}} = \frac{1}{c^2} \int_V \mathbf{r} \times \mathbf{S} dV$$

where

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}.$$

The simplified calculation below should therefore be understood as the lowest-order geometrical approximation to this full field integral.

6.2 Inner and Outer Contributions

The finite minor radius of the supertorus requires the inner and outer energy flows to be considered separately.

Their characteristic radii are

$$r_{\text{in}} = R - a, \quad r_{\text{out}} = R + a.$$

From the synchronous phase condition developed previously,

$$v_{\text{in}} = \omega(R - a)$$

and

$$v_{\text{out}} = \omega(R + a)$$

For the first-order model, we assume that the electromagnetic energy associated with the two effective branches is equal:

$$U_{\text{in}} = U_{\text{out}} = \frac{U}{2}.$$

This is an explicit simplifying assumption rather than a result of the model. It will ultimately have to be replaced by integration over the actual electromagnetic energy distribution.

The two branches belong to opposite sides of the local toroidal circulation. Their contributions to the global angular momentum therefore enter with opposite orientation in this reduced two-branch representation. Thus,

$$J = \frac{U}{2c^2} (r_{\text{out}}v_{\text{out}} - r_{\text{in}}v_{\text{in}}).$$

Substituting the radii and velocities gives

$$J = \frac{U\omega}{2c^2} [(R+a)^2 - (R-a)^2].$$

The geometrical difference simplifies to

$$(R+a)^2 - (R-a)^2 = 4Ra$$

Therefore,

$$J = \frac{2U\omega Ra}{c^2}$$

6.3 Applying the Electron Rest Energy

The total internal electromagnetic energy has already been constrained to the electron rest energy,

$$U = m_e c^2$$

Substituting this into the angular-momentum expression gives

$$J = 2m_e \omega Ra$$

The phase-closure condition provides

$$\omega R = c$$

Consequently,

$$J = 2m_e (\omega R) a = 2m_e c a,$$

and therefore

$$J = 2m_e c a$$

An interesting feature of this result is that the major radius R has disappeared. Once the rest energy and phase-closure condition are imposed, the angular momentum directly constrains the minor radius.

6.4 Determination of the Minor Radius

The measured intrinsic angular momentum of the electron is

$$J = \frac{\hbar}{2}$$

Equating this to the angular momentum of the simplified supertorus model gives

$$2m_e c a = \frac{\hbar}{2}$$

Solving for a ,

$$a = \frac{\hbar}{4m_e c}$$

Since the major radius was obtained as

$$R = \frac{\hbar}{m_e c},$$

the ratio of the two geometrical scales becomes

$$\frac{a}{R} = \frac{1}{4}$$

or simply

$$a = \frac{R}{4}$$

Using

$$R \approx 3.86 \times 10^{-13} \text{ m},$$

the minor radius is

$$a \approx 9.65 \times 10^{-14} \text{ m}$$

Thus, within the assumptions of the reduced two-branch model, the electron angular momentum fixes the aspect ratio of the supertorus to approximately

$$R:a = 4:1$$

6.5 Inner and Outer Radii

With

$$a=R/4,$$

the characteristic inner radius becomes

$$r_{\text{in}} = R - \frac{R}{4} = \frac{3R}{4},$$

or

$$r_{\text{in}} = \frac{3}{4}R$$

Similarly,

$$r_{\text{out}} = R + \frac{R}{4} = \frac{5R}{4},$$

so that

$$r_{\text{out}} = \frac{5}{4}R$$

Numerically,

$$r_{\text{in}} \approx 2.90 \times 10^{-13} \text{ m}$$

and

$$r_{\text{out}} \approx 4.83 \times 10^{-13} \text{ m}$$

The ratio of the outer and inner path lengths is consequently

$$\frac{L_{\text{out}}}{L_{\text{in}}} = \frac{R+a}{R-a} = \frac{5}{3}.$$

6.6 Corresponding Phase Velocities

The same aspect ratio determines the local phase velocities. Since

$$\omega R = c,$$

the inner phase velocity is

$$v_{\text{in}} = \omega(R-a) = c \left(1 - \frac{1}{4}\right),$$

and therefore

$$v_{\text{in}} = \frac{3}{4}c$$

Similarly,

$$v_{\text{out}} = \omega(R+a) = c \left(1 + \frac{1}{4}\right),$$

giving

$$v_{\text{out}} = \frac{5}{4}c$$

Thus, in the reduced model,

$$v_{\text{in}} = 0.75c, \quad v(R) = c, \quad v_{\text{out}} = 1.25c.$$

As emphasized previously, these quantities describe the progression of the coherent phase pattern. In particular, $v_{\text{out}} > c$ must not be interpreted without further analysis as a superluminal transport velocity of electromagnetic energy or information. The actual energy flow is determined by the Poynting vector and must ultimately be calculated from the complete field configuration.

6.7 What Has Been Determined

At this point, the two characteristic geometrical dimensions of the minimal supertorus model have been fixed:

$$R = \frac{\hbar}{m_e c}$$

and

$$a = \frac{\hbar}{4m_e c}$$

Therefore,

$$\frac{a}{R} = \frac{1}{4}$$

However, the status of these two results is not identical. The major radius follows after introducing the electron rest energy together with the quantum energy-frequency relation $U = \hbar\omega$. The minor radius additionally follows from the reduced two-branch representation

of electromagnetic angular momentum and the assumption

$$U_{\text{in}} = U_{\text{out}} = \frac{U}{2}.$$

Thus, $a = R/4$ should at present be regarded as a **result of the minimal supertorus approximation**, not yet as an exact prediction of a complete electromagnetic field solution.

This distinction provides a useful test for the subsequent analysis. Once

$$R = \bar{\lambda}_C, \quad a = \frac{R}{4}$$

have been fixed, they should no longer be adjusted to reproduce further electron properties.

The next quantities to be calculated are therefore genuine consequences of this fixed minimal geometry. In particular, the known volume,

$$V = 2\pi^2 R a^2$$

together with the energy condition

$$U_{\text{EM}} = m_e c^2,$$

allows the characteristic internal electric and magnetic field amplitudes E_0 and B_0 to be estimated.

That provides the next independent consistency test of the supertorus model.

7. Internal Electric and Magnetic Field Amplitudes

With the two characteristic dimensions of the minimal supertorus model now fixed, the total volume of the structure can be estimated. Combined with the rest-energy condition, this makes it possible to obtain a first estimate of the characteristic amplitudes of the internal electric and magnetic fields.

The calculation in this section does not attempt to determine the detailed spatial field distribution. Instead, it asks a simpler question:

what characteristic field amplitudes are required if the electron rest energy is stored electromagnetically within the volume of the supertorus?

The dimensions obtained in the preceding sections are

$$R = \frac{\hbar}{m_e c} \approx 3.86 \times 10^{-13} \text{ m}$$

and

$$a = \frac{R}{4} \approx 9.65 \times 10^{-14} \text{ m}.$$

These values are treated as fixed throughout the following calculation.

7.1 Volume of the Supertorus

For an ideal torus, the volume is

$$V = 2\pi^2 R a^2$$

Substituting the values of R and a ,

$$V = 2\pi^2 (3.86 \times 10^{-13}) (9.65 \times 10^{-14})^2$$

gives approximately

$$V \approx 7.10 \times 10^{-38} \text{ m}^3$$

The volume is extremely small, and therefore storing the electron rest energy within this region requires a correspondingly large electromagnetic energy density.

7.2 Mean Electromagnetic Energy Density

The total energy is constrained to equal the electron rest energy,

$$U = m_e c^2$$

Numerically,

$$U \approx 8.19 \times 10^{-14} \text{ J}$$

The mean electromagnetic energy density is therefore

$$\bar{u} = \frac{U}{V}$$

Substitution gives

$$\bar{u} \approx \frac{8.19 \times 10^{-14}}{7.10 \times 10^{-38}}$$

and hence

$$\bar{u} \approx 1.15 \times 10^{24} \text{ J m}^{-3}.$$

This value represents the mean internal electromagnetic energy density required by the minimal geometry.

It should not be interpreted as a uniform energy density throughout the supertorus. The actual fields are expected to vary across the toroidal cross-section and with phase. The value $\bar{u}$ is therefore only the volume average.

7.3 Electric and Magnetic Contributions

The instantaneous electromagnetic energy density in vacuum is

$$u_{\text{EM}} = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}.$$

For the first estimate, we assume that the local electric and magnetic fields satisfy the vacuum-wave relation

$$B_0 = \frac{E_0}{c}$$

Using

$$c^2 = \frac{1}{\mu_0 \epsilon_0}$$

the maximum electric and magnetic energy densities are equal:

$$\frac{\epsilon_0 E_0^2}{2} = \frac{B_0^2}{2\mu_0}.$$

The assumption of equal electric and magnetic contributions is appropriate as a first estimate for an internally propagating electromagnetic mode. It is not yet a derivation from the complete supertorus field configuration.

7.4 Time-Averaged Energy Density

Let the local fields vary sinusoidally,

$$E(t) = E_0 \cos \psi$$

and

$$B(t) = B_0 \cos \psi,$$

where ψ represents the local phase.

Because

$$\langle \cos^2 \psi \rangle = \frac{1}{2},$$

the time-averaged electric energy density is

$$\langle u_E \rangle = \frac{\epsilon_0 E_0^2}{4}.$$

Similarly,

$$\langle u_B \rangle = \frac{B_0^2}{4\mu_0}.$$

Since the two contributions are equal,

$$\bar{u} = \langle u_E \rangle + \langle u_B \rangle = \frac{\epsilon_0 E_0^2}{2}.$$

This relation allows the characteristic electric-field amplitude to be estimated directly from the mean energy density.

7.5 Electric-Field Amplitude

Solving the preceding expression for E_0 ,

$$E_0 = \sqrt{\frac{2\bar{u}}{\epsilon_0}}.$$

Using

$$\bar{u} \approx 1.15 \times 10^{24} \text{ J m}^{-3}$$

and

$$\epsilon_0 \approx 8.85 \times 10^{-12} \text{ F m}^{-1},$$

we obtain

$$E_0 \approx \sqrt{\frac{2(1.15 \times 10^{24})}{8.85 \times 10^{-12}}}.$$

This gives

$$E_0 \approx 5.10 \times 10^{17} \text{ V m}^{-1}.$$

Thus, the minimal supertorus model requires an internal electric-field amplitude of order 10^{17} V/m.

7.6 Magnetic-Field Amplitude

Using the assumed local electromagnetic relation

$$\mathbf{B}_0 = c\mathbf{E}_0,$$

the magnetic-field amplitude becomes

$$B_0 = \frac{5.10 \times 10^{17}}{2.998 \times 10^8}.$$

Therefore,

$$B_0 \approx 1.70 \times 10^9 \text{ T}$$

The internal magnetic field required by the energy-density estimate is therefore of order 10^9 T.

Again, this is a characteristic internal amplitude. It is not a prediction that an external observer would measure such a magnetic field outside the electron.

7.7 RMS Field Values

For sinusoidal fields, the root-mean-square values are

$$E_{\text{rms}} = \frac{E_0}{\sqrt{2}}$$

and

$$B_{\text{rms}} = \frac{B_0}{\sqrt{2}}$$

Numerically,

$$E_{\text{rms}} \approx 3.61 \times 10^{17} \text{ V m}^{-1}$$

and

$$B_{\text{rms}} \approx 1.20 \times 10^9 \text{ T}$$

The distinction between amplitude and RMS values is important because the internal

displacement current depends on the instantaneous derivative of the electric field, whereas the stored energy depends on the time average of E^2 and B^2 .

7.8 Displacement-Current Density

The oscillating electric field produces a displacement-current density,

$$\mathbf{J}_D = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

For

$$E(t) = E_0 \cos(\omega t + \phi),$$

the amplitude of the displacement-current density is

$$J_{D0} = \epsilon_0 \omega E_0.$$

The angular frequency obtained previously is

$$\omega \approx 7.76 \times 10^{20} \text{ s}^{-1}$$

Therefore,

$$J_{D0} = (8.85 \times 10^{-12})(7.76 \times 10^{20})(5.10 \times 10^{17})$$

which gives approximately

$$J_{D0} \approx 3.50 \times 10^{27} \text{ A m}^{-2}$$

This very large value is a local current-density amplitude. It must not be confused with the total current through an arbitrary cross-section.

The corresponding total displacement current would require integration,

$$I_D = \int_A \mathbf{J}_D \cdot d\mathbf{A}$$

At present, the effective cross-sectional distribution of $\mathbf{J}_D$ is not sufficiently specified to evaluate this integral uniquely.

7.9 Why the Displacement Current Is Not the Persistent Magnetic Current

A crucial distinction now becomes apparent.

For a sinusoidal electric field,

$$E(t) = E_0 \cos \omega t,$$

the displacement current varies as

$$J_D(t) = -\varepsilon_0 \omega E_0 \sin \omega t.$$

Its time average over one complete period is therefore

$$\langle J_D \rangle = 0$$

This means that the amplitude J_{D0} cannot simply be interpreted as a steady current responsible for the persistent magnetic moment of the electron.

This observation is important because it separates two physical roles within the supertorus model.

The oscillating displacement current describes the internal electromagnetic dynamics. The persistent magnetic moment, however, must arise from a quantity whose circulation does not average to zero over an oscillation period.

7.10 Nonzero Mean Energy Flow

The situation is different for electromagnetic energy transport. The Poynting vector is

$$S = \frac{1}{\mu_0} E \times B$$

If E and B oscillate coherently and in phase,

$$E = E_0 \cos \psi, \quad B = B_0 \cos \psi,$$

then the magnitude of the energy flux contains

$$S \propto E_0 B_0 \cos^2 \psi.$$

Because

$$\langle \cos^2 \psi \rangle = \frac{1}{2},$$

the time-averaged energy flow is nonzero:

$$\langle S \rangle \neq 0$$

Thus, a sinusoidally oscillating electromagnetic field can support a persistent mean circulation

of energy even though its displacement current has zero mean.

This provides an important internal distinction in the supertorus model:

$$\langle J_D \rangle = 0, \quad \langle S \rangle \neq 0.$$

The first describes oscillatory electromagnetic current; the second permits persistent internal energy circulation and therefore electromagnetic angular momentum.

7.11 Status of the Field-Amplitude Estimate

The values

$$E_0 \approx 5.10 \times 10^{17} \text{ V m}^{-1}$$

and

$$B_0 \approx 1.70 \times 10^9 \text{ T}$$

are not additional fitted parameters. Once the minimal geometry

$$R = \frac{\hbar}{m_e c}, \quad a = \frac{R}{4}$$

and total energy

$$U = m_e c^2$$

are fixed, these amplitudes follow from the assumed mean electromagnetic energy density and the local relation $B_0 = E_0/c$.

They nevertheless remain **first-order estimates**, because three simplifying assumptions have been made: the energy has been represented by its volume average, electric and magnetic energies have been taken to be equal on average, and the local fields have been assumed to satisfy the vacuum-wave relation.

A complete electromagnetic solution may therefore modify the spatial values of E and B , particularly between the inner and outer regions of the supertorus. What is much more strongly constrained is the integral condition

$$\int_V u_{\text{EM}} dV = m_e c^2$$

The present calculation establishes the characteristic scale of the internal fields required to satisfy that condition.

It also reveals an important consequence for the next stage of the model. The oscillating displacement current cannot by itself be identified with the persistent magnetic moment of the electron. The next section must therefore distinguish the **oscillatory displacement current** from the **effective circulation of the net electric charge** associated with the phase-locked supertorus structure

8. Displacement Current and Circulating Charge

The previous section showed that the internal electromagnetic field of the supertorus produces a very large oscillating displacement-current density. However, a sinusoidal displacement current has zero time average and therefore cannot, by itself, account for a persistent magnetic moment.

This requires a clear distinction between two different quantities: the **oscillatory displacement current**, associated with the time variation of the electric field, and the **effective circulation of the net electric charge**, associated with the motion of a phase-locked charge distribution around the supertorus.

The first describes the internal electromagnetic oscillation. The second may provide the persistent current associated with the magnetic moment.

8.1 Oscillating Displacement Current

Maxwell's displacement-current density is

$$\mathbf{J}_D = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

For a sinusoidally varying electric field,

$$E(t) = E_0 \cos(\omega t + \phi)$$

the displacement current is

$$J_D(t) = -\epsilon_0 \omega E_0 \sin(\omega t + \phi)$$

Its amplitude is therefore

$$J_{D0} = \epsilon_0 \omega E_0$$

while its mean value over one complete period is

$$\langle J_D \rangle = 0$$

Thus, the displacement current oscillates locally in both directions. Its maximum amplitude can be extremely large, as estimated in the preceding section, but this amplitude must not be interpreted as a steady current.

The distinction between **maximum current** and **time-averaged current** is essential. For a sinusoidal displacement current, the RMS value is nonzero,

$$J_{D,\text{rms}} = \frac{J_{D0}}{\sqrt{2}}$$

but its signed time average remains zero.

Therefore neither J_{D0} nor $J_{D,\text{rms}}$ can simply be substituted into the conventional current-loop expression for a permanent magnetic moment.

8.2 Traveling Phase Pattern

The individual toroidal cells oscillate at the same frequency but with a systematic phase shift. Consequently, although the displacement current at every fixed position changes sign, its **phase pattern travels continuously around the supertorus**.

Let ϕ denote the angular position around the major circumference. The fundamental phase-locked mode can be represented schematically by

$$J_D(\phi, t) = J_{D0} \sin(\phi - \omega t)$$

At every fixed position,

$$\langle J_D(\phi, t) \rangle_t = 0$$

but a point of constant phase satisfies

$$\varphi - \omega t = \text{constant}$$

Differentiating gives

$$\frac{d\varphi}{dt} = \omega$$

and therefore the local tangential phase velocity at radius r is

$$v_\varphi(r) = \omega r$$

This is an important distinction. The **local displacement current oscillates**, while the **phase pattern circulates continuously**.

8.3 Net Electron Charge as a Boundary Condition

The electron possesses the measured net electric charge

$$Q = -e$$

At the present stage, the supertorus model does not derive the value e from the internal electromagnetic structure. The measured electron charge is therefore introduced as an experimental boundary condition.

If $\rho(r,t)$ represents an effective charge distribution associated with the supertorus, it must satisfy

$$\int_V \rho(\mathbf{r}, t) dV = -e$$

The constancy of the total charge does not require the spatial distribution itself to remain stationary.

A phase-locked distribution may have the general form

$$\rho(\varphi, t) = \rho(\varphi - \omega t)$$

Such a distribution retains its shape while progressing around the supertorus with the phase pattern.

This is the central working hypothesis introduced in this section: **the net charge is associated with the closed electromagnetic**

structure and may be phase-locked to its circulating pattern.

It should not yet be interpreted as a derivation of charge from Maxwell's equations.

8.4 Effective Current of the Circulating Charge

If a charge density moves tangentially with velocity

$$v_\varphi(r) = \omega r$$

the corresponding effective charge-current density is

$$\mathbf{J}_Q = \rho \mathbf{v}$$

or, for the tangential component,

$$J_\varphi(r) = \rho(r)\omega r$$

This quantity is fundamentally different from the displacement current.

For the simplest idealization, suppose that the complete charge magnitude e completes one revolution around the supertorus during the period T . The corresponding effective current magnitude is then

$$I_Q = \frac{e}{T}$$

Since

$$T = \frac{2\pi}{\omega}$$

we obtain

$$I_Q = \frac{e\omega}{2\pi}$$

Using the previously obtained frequency,

$$\omega \approx 7.76 \times 10^{20} \text{ s}^{-1}$$

the corresponding current scale is approximately

$$I_Q \approx 19.8 \text{ A}$$

This value should be interpreted only as the effective current associated with one circulation of the full charge e per oscillation

period. The actual magnetic moment depends on the radial distribution of that charge.

8.5 Three Different Internal Quantities

The model now contains three different forms of internal circulation that should be kept conceptually separate.

The first is the displacement current,

$$\mathbf{J}_D = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

with

$$\langle J_D \rangle = 0$$

The second is the electromagnetic energy flow described by the Poynting vector,

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}.$$

Because $\mathbf{S}$ contains the product of two oscillating fields, a coherent mode may have

$$\langle S \rangle \neq 0$$

The third quantity is the effective current of the circulating net charge,

$$\mathbf{J}_Q = \rho \mathbf{v}.$$

For a phase-locked rotating charge distribution, this may also have a persistent circulation.

The conceptual separation can therefore be summarized as

$$\langle \mathbf{J}_D \rangle = 0, \quad \langle \mathbf{S} \rangle \neq 0, \quad \langle \mathbf{J}_Q \rangle \neq 0.$$

In the minimal model, these quantities have different physical roles:

displacement current $\longrightarrow$ internal electromagnetic oscillation

Poynting flow $\longrightarrow$ energy circulation and angular momentum

circulating charge $\longrightarrow$ magnetic moment

These identifications remain hypotheses of the reduced model and must ultimately be tested against a complete field solution.

8.6 Inner and Outer Charge Motion

The finite cross-section of the supertorus makes the distinction between its inner and outer regions particularly important.

Their characteristic radii are

$$r_{\text{in}} = R - a, \quad r_{\text{out}} = R + a.$$

Their phase velocities are

$$v_{\text{in}} = \omega(R - a)$$

and

$$v_{\text{out}} = \omega(R + a)$$

For the minimal geometry derived in Section 6,

$$a = \frac{R}{4}$$

these become

$$v_{\text{in}} = \frac{3}{4}c$$

and

$$v_{\text{out}} = \frac{5}{4}c$$

As in the preceding sections, these are phase velocities of the coherent pattern. They are not automatically velocities of individual charged material elements or of information.

This qualification is particularly important here. The assumption that an effective charge distribution is phase-locked to the electromagnetic pattern does **not yet prove that physical charge is transported locally at $\mathbf{v}_\Phi = \omega \mathbf{r}$** . A complete field model must determine the relation between the phase motion and the effective charge-current density.

8.7 Inner and Outer Contributions Need Not Be Equal

The total electron charge fixes only

$$\int_V \rho \, dV = -e$$

It does not determine how that charge is distributed across the toroidal cross-section.

In general,

$$\rho_{\text{in}} \neq \rho_{\text{out}}.$$

This is particularly relevant because the magnetic effect of circulating charge depends on radius. Charge located farther from the central axis contributes more strongly to the magnetic moment than the same amount of charge circulating at a smaller radius.

The inner and outer regions must therefore be evaluated separately rather than replaced by a single current at radius R.

This is also consistent with the geometry developed earlier: although the inner and outer portions of an individual toroidal cell can belong to the same synchronous oscillation, their path lengths, local phase velocities, effective volumes, and field densities are different.

8.8 Magnetic Moment as the Next Independent Test

The total charge e is now treated as an experimental boundary condition, while R , a , and ω have already been fixed by the preceding construction.

The remaining unknown is the detailed radial charge distribution.

The magnetic moment of a general current distribution is

$$\boldsymbol{\mu} = \frac{1}{2} \int_V \mathbf{r} \times \mathbf{J}_Q dV.$$

Substituting

$$\mathbf{J}_Q = \rho \mathbf{v}$$

gives

$$\boldsymbol{\mu} = \frac{1}{2} \int_V \rho \mathbf{r} \times \mathbf{v} dV.$$

For circular tangential motion with

$$\mathbf{v} = \omega \mathbf{r},$$

the magnitude reduces formally to

$$\mu = \frac{\omega}{2} \int_V \rho r^2 dV.$$

This final expression is particularly important. It shows explicitly that the magnetic moment does not depend only on the total charge,

$$\int_V \rho dV = -e,$$

but on the **second radial moment of the charge distribution**,

$$\int_V \rho r^2 dV$$

Consequently, the measured electron magnetic moment provides information about where, within the supertorus cross-section, the effective charge must be concentrated.

The next section, **9. Magnetic Moment of the Supertorus**, can therefore make a particularly strong test: instead of simply inserting μ_B , we can first calculate what magnetic moment follows from the already fixed geometry and a symmetric charge distribution, compare it with

$$\mu_B = \frac{e\hbar}{2m_e}$$

and only then ask what inner–outer redistribution of charge would be required to reproduce the measured electron magnetic moment.

9. Magnetic Moment of the Supertorus

The magnetic moment provides a particularly useful test of the supertorus model because the principal geometrical quantities have already been fixed. The major radius R , minor radius a , angular frequency ω , and total charge e should therefore not be adjusted merely to reproduce the magnetic moment.

The question is instead whether the circulating charge in this already constrained geometry naturally produces a magnetic moment of the correct magnitude.

For the electron, the natural reference value is the Bohr magneton,

$$\mu_B = \frac{e\hbar}{2m_e}$$

Numerically,

$$\mu_B \approx 9.274 \times 10^{-24} \text{ J T}^{-1}$$

The measured magnitude of the electron magnetic moment is slightly larger than μ_B . Before considering this small anomalous contribution, however, it is useful to determine what the simplest supertorus geometry predicts.

9.1 Magnetic Moment of a Circulating Charge

For a charge q circulating with period T , the current magnitude is

$$I = \frac{q}{T}$$

Since

$$T = \frac{2\pi}{\omega}$$

the current is

$$I = \frac{q\omega}{2\pi}$$

For a circular current loop of radius r , the magnetic moment magnitude is

$$\mu = I\pi r^2$$

Substituting the circulating current gives

$$\mu = \frac{q\omega r^2}{2}$$

This is the discrete-loop form of the more general expression derived in the preceding section,

$$\mu = \frac{\omega}{2} \int_V \rho r^2 dV$$

The magnetic moment is therefore determined by the second radial moment of the circulating charge distribution.

9.2 Effective Radius Required by the Bohr Magnetron

Before specifying a detailed charge distribution, we can ask what effective circulation radius would reproduce the Bohr magneton.

Let r_μ denote the effective magnetic radius. Then

$$\mu = \frac{e\omega r_\mu^2}{2}$$

Setting

$$\mu = \mu_B$$

gives

$$\frac{e\omega r_\mu^2}{2} = \frac{e\hbar}{2m_e}$$

After cancelling $e/2$,

$$\omega r_\mu^2 = \frac{\hbar}{m_e}$$

From the previous sections,

$$\omega = \frac{c}{R}$$

and

$$R = \frac{\hbar}{m_e c}$$

Therefore,

$$\frac{\hbar}{m_e} = cR$$

The magnetic-moment condition becomes

$$\frac{c}{R} r_\mu^2 = cR$$

Consequently,

$$r_\mu^2 = R^2$$

and hence

$$r_\mu = R$$

This is a remarkably simple result: **the Bohr magneton corresponds exactly to an effective charge-circulation radius equal to the major radius of the supertorus.**

No new length scale is required.

9.3 Direct Recovery of the Bohr Magnetron

The same result can be seen even more directly.

If the charge magnitude e effectively circulates at radius R , then

$$\mu = \frac{e\omega R^2}{2}$$

Using the phase-closure condition

$$\omega R = c,$$

we obtain

$$\mu = \frac{ecR}{2}$$

The major radius is

$$R = \frac{\hbar}{m_e c}.$$

Substitution gives

$$\mu = \frac{ec}{2} \frac{\hbar}{m_e c}$$

Cancelling c ,

$$\mu = \frac{e\hbar}{2m_e}$$

Therefore,

$$\mu = \mu_B$$

Within this effective-radius description, the already determined supertorus scale reproduces the Bohr magneton without introducing another dimensional parameter.

This agreement is noteworthy, although it is not yet a fully independent prediction: the

radius R itself was obtained using $U = \hbar\omega$. What is nontrivial is that the same radius also provides the correct magnetic-moment scale when combined with the measured charge e .

9.4 Why the Finite Minor Radius Matters

The actual supertorus is not an infinitesimally thin ring at $r=R$. Its charge occupies a finite cross-section extending approximately from

$$r_{\text{in}} = R - a = \frac{3R}{4}$$

to

$$r_{\text{out}} = R + a = \frac{5R}{4}$$

The magnetic moment therefore depends on the actual radial distribution of charge.

Define the charge-weighted mean-square radius by

$$\langle r^2 \rangle_Q = \frac{1}{e} \int_V |\rho| r^2 dV$$

For a charge distribution of one sign, the magnetic-moment magnitude becomes

$$|\mu| = \frac{e\omega}{2} \langle r^2 \rangle_Q$$

Using the Bohr magneton as reference,

$$\frac{|\mu|}{\mu_B} = \frac{\langle r^2 \rangle_Q}{R^2}$$

This equation is particularly useful because it converts the magnetic-moment problem directly into a geometrical constraint on the radial charge distribution.

To reproduce exactly the Bohr magneton,

$$\langle r^2 \rangle_Q = R^2$$

9.5 A Uniform Toroidal Cross-Section Does Not Give Exactly μ_B

This point provides an important test.

Suppose, as a first approximation, that the effective charge is uniformly distributed throughout the toroidal volume. Introduce a

local cross-sectional radius s and angle θ , so that the distance from the major axis is

$$r = R + s \cos \theta.$$

with

$$0 \leq s \leq a.$$

For a uniform volume distribution in a torus, the charge-weighted mean-square radius is

$$\langle r^2 \rangle_Q = R^2 + \frac{3a^2}{4}$$

With

$$a = \frac{R}{4},$$

this becomes

$$\langle r^2 \rangle_Q = R^2 \left(1 + \frac{3}{64} \right).$$

Therefore,

$$\langle r^2 \rangle_Q = \frac{67}{64} R^2$$

The predicted magnetic moment would then be

$$\mu_{\text{uniform}} = \frac{67}{64} \mu_B$$

or numerically,

$$\mu_{\text{uniform}} \approx 1.046875 \mu_B$$

Thus, a uniform volume distribution would produce a magnetic moment approximately 4.69% larger than the Bohr magneton.

This is useful rather than problematic: it shows that the magnetic moment is sensitive enough to constrain the internal charge distribution. The charge cannot simply be assumed to fill the toroidal volume uniformly if the minimal model is to reproduce the electron magnetic moment.

9.6 The Measured Electron Magnetic Moment

The actual electron magnetic moment differs slightly from the Bohr magneton. In magnitude it may be written as

$$|\mu_e| = \frac{g_e}{2} \mu_B$$

where g_e is the electron g-factor.

Equivalently,

$$|\mu_e| = (1 + a_e) \mu_B$$

where

$$a_e = \frac{g_e - 2}{2}$$

is the electron magnetic anomaly.

Its magnitude is approximately

$$a_e \approx 1.16 \times 10^{-3}$$

Therefore,

$$|\mu_e| \approx 1.00116 \mu_B$$

The supertorus relation

$$\frac{|\mu|}{\mu_B} = \frac{\langle r^2 \rangle_Q}{R^2}$$

then requires

$$\langle r^2 \rangle_Q = (1 + a_e) R^2$$

Thus, the measured magnetic moment corresponds to an effective magnetic radius

$$r_{\mu,e} = R \sqrt{1 + a_e}$$

Since $a_e \ll 1$,

$$r_{\mu,e} \approx R \left(1 + \frac{a_e}{2} \right)$$

Numerically,

$$r_{\mu,e} \approx 1.00058 R$$

The experimentally measured magnetic moment therefore corresponds to an effective charge radius only about 0.058% larger than R .

9.7 Inner and Outer Contributions

This result gives a useful way to examine the inner and outer parts separately.

Suppose, as a deliberately simplified two-radius approximation, that a fraction f of the charge magnitude circulates at the outer radius

$$r_{\text{out}} = \frac{5}{4}R$$

and the remaining fraction $1 - f$ circulates at the inner radius

$$r_{\text{in}} = \frac{3}{4}R.$$

The charge-weighted mean-square radius is then

$$\langle r^2 \rangle_Q = f \left(\frac{5}{4}R \right)^2 + (1 - f) \left(\frac{3}{4}R \right)^2$$

Dividing by R^2 ,

$$\frac{\langle r^2 \rangle_Q}{R^2} = \frac{25}{16}f + \frac{9}{16}(1 - f)$$

This simplifies to

$$\frac{\langle r^2 \rangle_Q}{R^2} = \frac{9}{16} + f$$

For the Bohr magneton, the required condition is

$$\frac{9}{16} + f = 1,$$

Giving

$$f = \frac{7}{16} = 0.4375$$

Thus, in this crude two-radius approximation, 43.75% of the effective circulating charge would lie on the outer branch and 56.25% on the inner branch to reproduce exactly μ_B .

For the measured electron magnetic moment,

$$\frac{9}{16} + f = 1 + a_e$$

Therefore,

$$f = \frac{7}{16} + a_e$$

or approximately

$$f \approx 0.43866$$

The corresponding inner fraction is

$$1 - f \approx 0.56134$$

These fractions should **not** be interpreted as a derived microscopic charge distribution. The two-radius representation is much too coarse for that. Its value is that it demonstrates quantitatively how the measured magnetic moment constrains the balance between inner and outer contributions.

9.8 A More General Constraint on the Charge Distribution

The result can be stated without making the two-radius approximation.

For the electron magnetic moment, the charge distribution must satisfy

$$\frac{1}{e} \int_V |\rho| r^2 dV = (1 + a_e)R^2$$

Together with charge conservation,

$$\int_V |\rho| dV = e$$

these are two independent integral constraints on the internal charge distribution.

This is more useful than simply assuming a particular $\rho(r)$. Any future detailed electromagnetic solution of the supertorus must satisfy both conditions simultaneously.

In normalized form,

$$\left\langle \left(\frac{r}{R} \right)^2 \right\rangle_Q = 1 + a_e$$

This gives the magnetic moment a direct geometrical interpretation within the model.

9.9 Relation Between Magnetic Moment and Angular Momentum

An interesting structural comparison now appears.

The electron angular momentum used in Section 6 is

$$J = \frac{\hbar}{2},$$

while the leading magnetic moment is

$$\mu_B = \frac{e\hbar}{2m_e}.$$

Combining them gives

$$\frac{\mu_B}{J} = \frac{e}{m_e}$$

For the measured electron moment,

$$\frac{|\mu_e|}{J} = (1 + a_e) \frac{e}{m_e}$$

In conventional notation this corresponds to

$$|\mu_e| = g_e \frac{e}{2m_e} J$$

with

$$g_e = 2(1 + a_e)$$

Within the present model, however, the angular momentum and magnetic moment need not arise from identical spatial weightings. Angular momentum is associated with the circulating electromagnetic energy, whereas the magnetic moment is associated with the effective circulating charge. Their ratio can therefore contain information about the relative spatial distributions of **energy and charge** within the supertorus.

This distinction may be one of the most important structural features of the model.

9.10 Result of the Magnetic-Moment Test

The magnetic-moment calculation produces three increasingly restrictive results.

First, if the effective circulating charge is concentrated at the major radius R , the model gives exactly

$$\mu = \mu_B.$$

Second, a finite toroidal cross-section makes the magnetic moment sensitive to the radial

charge distribution. A uniform volume distribution would give

$$\mu_{\text{uniform}} = \frac{67}{64} \mu_B \approx 1.0469 \mu_B,$$

which is too large.

Third, the measured electron magnetic moment requires

$$\langle r^2 \rangle_Q = (1 + a_e) R^2$$

or equivalently

$$r_{\mu,e} \approx 1.00058 R.$$

The magnetic moment therefore does not merely supply another number that can be fitted by the model. It provides a quantitative constraint on the internal distribution of the charge.

At this point the minimal supertorus has acquired a notable hierarchy of constraints:

$$\begin{array}{l} m_e c^2 \longrightarrow \omega, R \\ \frac{\hbar}{2} \longrightarrow a \\ e \longrightarrow \text{total charge} \\ \mu_e \longrightarrow \langle r^2 \rangle_Q \end{array}$$

The next question is especially interesting: whether the same internal electromagnetic fields estimated in Section 7 can support the charge distribution required here, rather than treating charge and field structure as independent ingredients. That provides the natural bridge to the next stage of the quantitative consistency test.

10. Inner–Outer Asymmetry of the Charge Distribution

The magnetic-moment calculation of the preceding section showed that the finite cross-

section of the supertorus cannot be ignored. Charge located at different distances from the central axis contributes differently to the magnetic moment because the contribution is weighted by r^2 .

The measured electron charge fixes the total charge,

$$Q = -e$$

while the measured magnetic moment imposes the additional condition

$$\langle r^2 \rangle_Q = (1 + a_e)R^2$$

The charge distribution must therefore satisfy two constraints simultaneously: its integral must give the electron charge, while its radial second moment must reproduce the electron magnetic moment.

This section examines whether the required asymmetry between the inner and outer regions of the supertorus can arise naturally from its geometry.

10.1 Two-Region Approximation

As a first approximation, divide the charge distribution into two effective components: an inner component q_{in} located at radius

$$r_{\text{in}} = R - a$$

and an outer component q_{out} located at

$$r_{\text{out}} = R + a$$

The total charge magnitude must satisfy

$$q_{\text{in}} + q_{\text{out}} = e$$

For the minimal geometry,

$$a = \frac{R}{4},$$

so that

$$r_{\text{in}} = \frac{3}{4}R$$

and

$$r_{\text{out}} = \frac{5}{4}R$$

The magnetic moment of the two components is

$$|\mu| = \frac{\omega}{2} (q_{\text{in}} r_{\text{in}}^2 + q_{\text{out}} r_{\text{out}}^2).$$

Using

$$\mu_B = \frac{e\omega R^2}{2},$$

the ratio becomes

$$\frac{|\mu|}{\mu_B} = \frac{q_{\text{in}} r_{\text{in}}^2 + q_{\text{out}} r_{\text{out}}^2}{eR^2}.$$

10.2 Charge Fractions Required by the Electron Magnetic Moment

Introduce the outer charge fraction

$$f = \frac{q_{\text{out}}}{e}$$

Then

$$\frac{q_{\text{in}}}{e} = 1 - f$$

Using the inner and outer radii, the magnetic-moment ratio becomes

$$\frac{|\mu|}{\mu_B} = (1 - f) \left(\frac{3}{4} \right)^2 + f \left(\frac{5}{4} \right)^2.$$

Since

$$\left(\frac{3}{4} \right)^2 = \frac{9}{16}$$

And

$$\left(\frac{5}{4} \right)^2 = \frac{25}{16},$$

the expression simplifies to

$$\frac{|\mu|}{\mu_B} = \frac{9}{16} + f$$

For the measured electron magnetic moment,

$$\frac{|\mu_e|}{\mu_B} = 1 + a_e \approx 1.00115965$$

Therefore,

$$f = 1 + a_e - \frac{9}{16}$$

Numerically,

$$f \approx 0.43866$$

Thus,

$$q_{\text{out}} \approx 0.43866 e$$

And

$$q_{\text{in}} \approx 0.56134 e$$

In this simplified two-region representation, approximately 56.1% of the effective circulating charge is associated with the inner region and 43.9% with the outer region.

The ratio is

$$\frac{q_{\text{in}}}{q_{\text{out}}} \approx 1.280$$

The inner region therefore carries approximately 28% more effective charge than the outer region in this reduced representation.

10.3 Why More Charge Can Occur on the Inner Side

At first sight, the result may appear surprising because the outer region has a larger circumference. However, the supertorus is a three-dimensional coherent electromagnetic structure, not a uniform material ring.

The inner and outer regions differ in several respects:

- their path lengths are different;
- their local phase velocities are different;
- the available volume per phase interval is different;
- the field amplitudes and displacement-current densities need not be identical.

In particular, the outer path is longer,

$$L_{\text{out}} = 2\pi(R + a)$$

while the inner path is

$$L_{\text{in}} = 2\pi(R - a)$$

For $a=R/4$,

$$\frac{L_{\text{out}}}{L_{\text{in}}} = \frac{5}{3}$$

Thus, the same phase interval occupies more spatial length on the outer side than on the inner side.

If comparable electromagnetic energy is associated with equal phase intervals, the energy density on the inner side can naturally be higher.

10.4 A Simple Geometrical Field-Scaling Estimate

A useful first estimate can be obtained by assuming that the available effective volume per phase interval increases approximately in proportion to the local major radius r .

Under that assumption,

$$u(r) \propto \frac{1}{r}$$

For an electromagnetic field,

$$u \propto E^2$$

so that

$$E(r) \propto \frac{1}{\sqrt{r}}$$

If the effective charge contribution of each phase-locked toroidal cell is approximately proportional to the local electric-field amplitude, then

$$q(r) \propto E(r)$$

and consequently

$$q(r) \propto r^{-1/2}$$

This is not yet derived from Maxwell's equations. It is a simple geometrical scaling hypothesis that can be tested against the charge asymmetry required by the magnetic moment.

10.5 Geometrically Predicted Inner–Outer Ratio

Using

$$q(r) \propto \frac{1}{\sqrt{r}},$$

the inner-to-outer charge ratio becomes

$$\frac{q_{\text{in}}}{q_{\text{out}}} = \sqrt{\frac{r_{\text{out}}}{r_{\text{in}}}}$$

Since

$$\frac{r_{\text{out}}}{r_{\text{in}}} = \frac{5/4}{3/4} = \frac{5}{3},$$

we obtain

$$\frac{q_{\text{in}}}{q_{\text{out}}} = \sqrt{\frac{5}{3}} \approx 1.291.$$

This may be compared with the ratio required by the measured magnetic moment,

$$\left(\frac{q_{\text{in}}}{q_{\text{out}}}\right)_{\mu_e} \approx 1.280$$

The difference is less than approximately 1%.

This agreement is interesting because the geometrical estimate was obtained without using the measured magnetic moment in the scaling relation.

10.6 Normalized Geometrical Charge Fractions

Normalizing the geometrical relation so that

$$q_{\text{in}} + q_{\text{out}} = e,$$

gives

$$\frac{q_{\text{in}}}{e} = \frac{\sqrt{5/3}}{1 + \sqrt{5/3}}.$$

Numerically,

$$q_{\text{in}} \approx 0.5635 e$$

Similarly,

$$q_{\text{out}} \approx 0.4365 e$$

The comparison is therefore

$$q_{\text{in}} : 0.5635 e \quad (\text{geometrical estimate})$$

versus

$$q_{\text{in}} : 0.5613 e \quad (\text{electron magnetic moment}).$$

For the outer region,

$$q_{\text{out}} : 0.4365 e \quad (\text{geometrical estimate})$$

versus

$$q_{\text{out}} : 0.4387 e \quad (\text{electron magnetic moment}).$$

The difference between the two estimates is small.

10.7 Magnetic Moment Predicted by the Geometrical Asymmetry

The geometrical charge fractions can now be inserted into the two-region magnetic-moment expression,

$$\frac{\mu_{\text{geom}}}{\mu_B} = 0.5635 \left(\frac{3}{4}\right)^2 + 0.4365 \left(\frac{5}{4}\right)^2.$$

This gives approximately

$$\frac{\mu_{\text{geom}}}{\mu_B} \approx 0.9990$$

The measured electron value is

$$\frac{|\mu_e|}{\mu_B} \approx 1.00116$$

Thus, the simple geometrical scaling model differs from the measured electron magnetic moment by only about

$$0.22\%.$$

This numerical proximity should not be interpreted as a proof of the proposed charge distribution. The scaling

$$E(r) \propto r^{-1/2}$$

was introduced as a first geometrical approximation rather than derived from a complete electromagnetic field solution.

Nevertheless, the result is useful because it gives both the correct **direction of the asymmetry** and approximately the required **magnitude of the asymmetry** without directly fitting the magnetic moment.

10.8 From Two Regions to a Continuous Cross-Section

The division into only an inner and an outer charge component is clearly an approximation. A real supertorus would possess a continuous charge and field distribution across its cross-section.

A convenient local coordinate is the poloidal angle θ , for which the distance from the central axis can be written as

$$r(\theta) = R + a \cos \theta.$$

With

$$a = \frac{R}{4},$$

this becomes

$$r(\theta) = R \left(1 + \frac{1}{4} \cos \theta \right).$$

The corresponding first-order field-amplitude scaling would be

$$E(\theta) \propto \frac{1}{\sqrt{R + a \cos \theta}}.$$

A future continuous model should determine the charge element dq from this field distribution and then evaluate the magnetic moment from

$$\mu = \frac{\omega}{2} \int r^2 dq$$

This would replace the two-region approximation by an integral over the complete toroidal cross-section.

10.9 Important Limitation of the Present Asymmetry Model

The present argument does **not** yet derive electric charge from electromagnetic fields.

The relation

$$q(r) \propto E(r)$$

is a phenomenological scaling assumption used only to test whether the geometry can naturally produce an inner–outer asymmetry of the required order.

A complete theory would have to determine the charge distribution from the actual electromagnetic field structure and boundary conditions.

Therefore, the result

$$q_{\text{in}} \approx 0.56 e, \quad q_{\text{out}} \approx 0.44 e$$

should be interpreted as a **constraint on a future field solution**, not as an established microscopic division of the electron charge.

Any successful solution should reproduce approximately the same charge-weighted radial moment,

$$\langle r^2 \rangle_Q \approx 1.00116 R^2$$

10.10 Result of the Inner–Outer Asymmetry Test

The analysis leads to three useful conclusions.

First, the observed magnetic moment requires the effective charge distribution to be slightly asymmetric across the supertorus cross-section.

In the two-region approximation,

$$q_{\text{in}} \approx 0.5613 e, \quad q_{\text{out}} \approx 0.4387 e$$

Second, a simple geometrical argument based on a larger available volume on the outer side gives

$$\langle r^2 \rangle_Q \approx 1.00116 R^2.$$

which predicts approximately

$$q_{\text{in}} \approx 0.5635 e, \quad q_{\text{out}} \approx 0.4365 e$$

Third, inserting this geometrically estimated asymmetry into the magnetic-moment calculation gives

$$\mu_{\text{geom}} \approx 0.9990 \mu_B,$$

which lies close to the measured value

$$|\mu_e| \approx 1.00116 \mu_B.$$

The remaining discrepancy is small compared with the simplicity of the approximation, but it should not be eliminated by introducing an arbitrary correction factor. Instead, it becomes a quantitative target for a more complete field calculation.

The value of this result is therefore not that the anomalous magnetic moment has already been explained. Rather, the geometry of the supertorus appears capable of producing an inner–outer asymmetry of approximately the magnitude required by the observed magnetic moment.

The next section can now collect all constraints obtained so far—rest energy, charge, spin, magnetic moment, phase closure, and geometrical dimensions—into a single framework and distinguish clearly which quantities were used as inputs and which emerged from the minimal supertorus model.

11. A Unified Set of Electron Constraints

The preceding sections introduced the electron properties one by one as constraints on the supertorus geometry and field structure. At this stage, it is useful to collect them into a single framework.

The purpose is not merely organizational. A model becomes more restrictive when the same internal geometry must satisfy several

experimentally known properties simultaneously. The supertorus is therefore judged not by one successful numerical coincidence, but by whether the same set of parameters can remain compatible with rest energy, charge, angular momentum, magnetic moment, phase closure, and internal electromagnetic energy.

11.1 Rest Energy

The total electromagnetic energy of the supertorus is constrained by the electron rest energy,

$$U_{\text{EM}} = m_e c^2$$

In full field form,

$$\int_V \left(\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right) dV = m_e c^2$$

This condition fixes the total amount of electromagnetic energy, but not by itself its spatial distribution.

11.2 Energy-Frequency Boundary Condition

The model uses the experimentally established energy-frequency relation

$$U = \hbar \omega$$

Combined with the electron rest energy,

$$m_e c^2 = \hbar$$

which gives

$$\omega = \frac{m_e c^2}{\hbar}$$

This relation is an imposed boundary condition, not an independent prediction of the supertorus model.

11.3 Phase Closure

For the fundamental mode, one complete phase cycle is distributed around the major circumference,

$$k(2\pi R) = 2$$

Therefore,

$$k = \frac{1}{R}$$

Using the characteristic electromagnetic relation

$$\omega = ck$$

we obtain

$$\omega R = c$$

Combining this with the energy-frequency boundary condition fixes the major radius:

$$R = \frac{\hbar}{m_e c} = \overline{\lambda}_c$$

Thus, the major radius of the minimal supertorus is fixed at the reduced Compton scale.

11.4 Angular Momentum

The intrinsic angular momentum of the electron provides the next constraint,

$$J = \frac{\hbar}{2}$$

Within the reduced inner-outer energy-flow model, the electromagnetic angular momentum was obtained as

$$J = \frac{U\omega}{2c^2} [(R+a)^2 - (R-a)^2]$$

Since

$$(R+a)^2 - (R-a)^2 = 4Ra$$

this becomes

$$J = \frac{2U\omega Ra}{c^2}$$

Using

$$U = m_e c^2$$

and

$$\omega R = c,$$

we obtain

$$J = 2m_e ca$$

Equating this to the electron spin gives

$$2m_e ca = \frac{\hbar}{2}$$

and therefore

$$a = \frac{\hbar}{4m_e c}$$

Since

$$R = \frac{\hbar}{m_e c},$$

the aspect ratio is

$$\boxed{\frac{a}{R} = \frac{1}{4}.$$

Thus, in the minimal two-branch approximation,

$$\boxed{R : a = 4 : 1.}$$

11.5 Inner and Outer Radii and Phase Velocities

The fixed aspect ratio gives

$$r_{\text{in}} = \frac{3}{4}R, \quad r_{\text{out}} = \frac{5}{4}R$$

The synchronous phase condition then gives

$$v_{\text{in}} = \frac{3}{4}c$$

and

$$v_{\text{out}} = \frac{5}{4}c$$

These quantities represent local phase velocities of the coherent pattern, not automatically local transport velocities of energy or information.

11.6 Internal Field Amplitudes

With both R and a fixed, the ideal toroidal volume is

$$V = 2\pi^2 R a^2$$

Numerically,

$$V \approx 7.10 \times 10^{-38} \text{ m}^3$$

The mean electromagnetic energy density is therefore

$$\bar{u} = \frac{m_e c^2}{V} \approx 1.15 \times 10^{24} \text{ J m}^{-3}$$

Assuming equal average electric and magnetic contributions and the local relation

$$B_0 = \frac{E_0}{c}$$

the characteristic field amplitudes are

$$E_0 \approx 5.10 \times 10^{17} \text{ V m}^{-1}$$

and

$$B_0 \approx 1.70 \times 10^9 \text{ T}$$

These remain first-order average-field estimates rather than a complete local field solution.

11.7 Displacement Current

The internal electric oscillation produces the displacement-current density

$$\mathbf{J}_D = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

Its characteristic amplitude is

$$J_{D0} = \epsilon_0 \omega E_0 \approx 3.50 \times 10^{27} \text{ A m}^{-2}$$

However,

$$\langle J_D \rangle = 0$$

Thus, this current describes the oscillatory electromagnetic dynamics but is not identified directly with the persistent magnetic current.

11.8 Net Electric Charge

The electron charge provides an independent boundary condition,

$$Q = -e$$

The effective charge density must satisfy

$$\int_V \rho \, dV = -e$$

The detailed spatial function $\rho(r)$ remains undetermined by the total charge alone.

11.9 Magnetic Moment

For a phase-locked rotating charge distribution,

$$\mu = \frac{\omega}{2} \int_V \rho r^2 \, dV$$

If the charge has an effective circulation radius R , then

$$\mu = \frac{e\omega R^2}{2}$$

Using

$$\omega R = c$$

and

$$R = \frac{\hbar}{m_e c},$$

we obtain

$$\mu = \frac{e\hbar}{2m_e} = \mu_B$$

Thus, the same major radius that follows from the energy and phase conditions also gives the correct leading magnetic-moment scale.

11.10 Constraint from the Measured Electron Magnetic Moment

The measured electron magnetic moment is

$$|\mu_e| = (1 + a_e)\mu_B$$

with

$$a_e \approx 1.16 \times 10^{-3}$$

Therefore the radial charge distribution must satisfy

$$\langle r^2 \rangle_Q = (1 + a_e)R^2$$

In the simplified two-region approximation, this corresponds to

$$q_{\text{in}} \approx 0.5613 \, e$$

and

$$q_{\text{out}} \approx 0.4387 \, e$$

A simple geometrical scaling based on

$$E(r) \propto r^{-1/2}$$

produces approximately

$$q_{\text{in}} \approx 0.5635 e, \quad q_{\text{out}} \approx 0.4365 e$$

The close numerical agreement is suggestive, but the field scaling remains an approximation and should not yet be interpreted as a derivation of the electron charge distribution.

11.11 Unified Constraint Structure

The minimal supertorus model can now be summarized by the following chain:

$$\begin{aligned} m_e c^2 \text{ and } U = \hbar \omega &\implies \omega \\ \omega R = c &\implies R = \frac{\hbar}{m_e c} \\ J = \frac{\hbar}{2} &\implies a = \frac{R}{4} \\ R, a, m_e c^2 &\implies E_0, B_0 \\ Q = -e &\implies \int_V \rho dV = -e \\ \mu_e &\implies \langle r^2 \rangle_Q \end{aligned}$$

The resulting minimal parameter set is therefore approximately

$$\begin{aligned} R &\approx 3.86 \times 10^{-13} \text{ m}, \\ a &\approx 9.65 \times 10^{-14} \text{ m}, \\ \omega &\approx 7.76 \times 10^{20} \text{ s}^{-1}, \\ E_0 &\approx 5.10 \times 10^{17} \text{ V m}^{-1}, \\ B_0 &\approx 1.70 \times 10^9 \text{ T}. \end{aligned}$$

These quantities all refer to the same minimal geometry.

11.12 The Main Consistency Requirement

The most important result of the analysis is not any single numerical value. It is the requirement that **one and the same field**

configuration must satisfy all constraints simultaneously.

A complete supertorus solution must therefore satisfy, at minimum,

$$\begin{aligned} \int_V u_{\text{EM}} dV &= m_e c^2, \\ \mathbf{J}_{\text{EM}} &= \frac{1}{c^2} \int_V \mathbf{r} \times \mathbf{S} dV, \quad |\mathbf{J}_{\text{EM}}| = \frac{\hbar}{2}, \\ \int_V \rho dV &= -e, \end{aligned}$$

and

$$\boldsymbol{\mu} = \frac{1}{2} \int_V \mathbf{r} \times \mathbf{J}_Q dV = \boldsymbol{\mu}_e.$$

In addition, the structure must satisfy the non-radiation condition developed in Section 4,

$$\lim_{r \rightarrow \infty} \oint_{A(r)} \langle \mathbf{S} \rangle \cdot d\mathbf{A} = 0.$$

These equations collectively define the target boundary-value problem for a complete electromagnetic supertorus.

11.13 What Has Been Achieved at the Minimal-Model Level

The calculations so far do not constitute a complete solution of that field problem. They do, however, reduce the range of possible structures considerably.

The major radius is no longer arbitrary.

The minor radius is no longer arbitrary within the two-branch angular-momentum approximation.

The characteristic internal frequency is fixed.

The characteristic field amplitudes have been estimated.

The net charge is fixed by experiment.

The measured magnetic moment constrains the radial charge distribution.

The model therefore has progressed from a qualitative toroidal hypothesis to a quantitatively constrained electromagnetic configuration.

The remaining task is to distinguish clearly between quantities that have been **assumed**, quantities that have been **used as experimental boundary conditions**, quantities that have been **derived within the minimal approximation**, and quantities that should eventually become genuine **predictions**.

That distinction is essential for evaluating the evidential strength of the model and is addressed explicitly in the next section, **12. What Is Derived and What Is Assumed**.

12. What Is Derived and What Is Assumed

The preceding sections combine experimentally known electron properties, geometrical assumptions, electromagnetic relations, and results obtained from the minimal supertorus model. To evaluate the significance of the model correctly, these categories must be kept separate.

A quantity that is inserted as a measured boundary condition cannot later be presented as an independent prediction. Likewise, a result obtained only after a simplifying geometrical assumption should be distinguished from a result that follows directly from a conservation law or an experimentally established relation.

The purpose of this section is therefore to make the logical structure of the model explicit.

12.1 Experimental Boundary Conditions

The following quantities are taken directly from the known properties of the electron.

The rest energy is

$$U = m_e c^2$$

The electric charge is

$$Q = -e$$

The intrinsic angular momentum is

$$J = \frac{\hbar}{2}$$

and the measured magnetic moment is

$$|\mu_e| = (1 + a_e)\mu_B$$

with

$$\mu_B = \frac{e\hbar}{2m_e}$$

These quantities are not predictions of the supertorus model. They are experimental properties that the model is required to reproduce.

12.2 The Energy-Frequency Relation

The relation

$$U = \hbar\omega$$

is also introduced as a boundary condition.

Combining it with the measured electron rest energy gives

$$\omega = \frac{m_e c^2}{\hbar}$$

Therefore, the characteristic frequency

$$\omega \approx 7.76 \times 10^{20} \text{ s}^{-1}$$

is not obtained from the electromagnetic geometry alone.

This point is especially important because the Compton scale subsequently emerges from this relation.

The basic supertorus construction introduces several assumptions that define the minimal model.

First, the internal electromagnetic field is assumed to form a closed sequence of coherent toroidal cells.

Second, neighboring cells are assumed to possess a systematic phase shift such that one complete circuit produces the phase change

$$\Delta\phi_{\text{total}} = 2\pi.$$

This is the fundamental phase-closure assumption.

For the continuous representation it gives

$$kR = 1$$

The model further uses the electromagnetic propagation relation

$$\omega = ck$$

Combining the two gives

$$\omega R = c$$

The relation $\omega R = c$ is therefore derived from the phase-closure geometry once the local dispersion relation $\omega = ck$ is assumed.

12.4 What Is Derived for the Major Radius

The major radius follows from

$$\omega R = c$$

together with

$$\omega = \frac{m_e c^2}{\hbar}.$$

Thus,

$$R = \frac{\hbar}{m_e c}$$

Therefore,

$$R = \overline{\lambda}_C$$

This result is derived within the model, but it is **not an independent prediction of the Compton scale**, because the quantum relation $U = \hbar\omega$ was already imposed.

The more precise statement is therefore:

Given the electron rest energy, the relation $U = \hbar\omega$, and one complete phase cycle around

the supertorus, the required major radius is the reduced Compton wavelength.

That is a consistency result rather than a prediction from classical electromagnetism alone.

12.5 Assumptions Entering the Minor Radius

The minor radius requires additional assumptions.

The reduced angular-momentum model treats the inner and outer regions separately and assumes equal effective electromagnetic energies,

$$U_{\text{in}} = U_{\text{out}} = \frac{U}{2}$$

It also assumes that the two effective branches contribute with opposite orientation to the global angular momentum.

Under these assumptions,

$$J = \frac{U\omega}{2c^2} [(R+a)^2 - (R-a)^2]$$

Using

$$U = m_e c^2$$

and

$$\omega R = c,$$

this reduces to

$$J = 2m_e c a$$

Applying the measured electron angular momentum

$$J = \frac{\hbar}{2}$$

gives

$$a = \frac{\hbar}{4m_e c} = \frac{R}{4}$$

Thus, the aspect ratio

$$\frac{a}{R} = \frac{1}{4}$$

is a derived result **within the minimal two-branch approximation**, not yet an exact result of the full electromagnetic field problem.

12.6 Internal Field Amplitudes

Once R and a are fixed, the volume

$$V = 2\pi^2 R a^2$$

is derived geometrically.

The mean electromagnetic energy density then follows from

$$\bar{u} = \frac{m_e c^2}{V}$$

However, the estimates of E_0 and B_0 require further assumptions.

The first is that electric and magnetic contributions are equal on average.

The second is the local relation

$$B_0 = \frac{E_0}{c}$$

Under these assumptions,

$$E_0 \approx 5.10 \times 10^{17} \text{ V m}^{-1}$$

and

$$B_0 \approx 1.70 \times 10^9 \text{ T}$$

These are therefore **derived estimates conditional on the assumed average field structure**.

12.7 What Is Derived About the Displacement Current

Once E_0 and ω have been estimated, Maxwell's displacement-current relation gives

$$J_{D0} = \epsilon_0 \omega E_0$$

The corresponding estimate is

$$J_{D0} \approx 3.50 \times 10^{27} \text{ A m}^{-2}$$

For sinusoidal oscillation,

$$\langle J_D \rangle = 0$$

This zero average is not an assumption; it follows directly from the assumed sinusoidal time dependence.

The conclusion that the persistent magnetic moment cannot simply be identified with the sinusoidal displacement current is therefore a derived consequence of the oscillatory model.

12.8 Assumption of Phase-Locked Charge Circulation

The total charge

$$Q = -e$$

is experimental input.

However, the hypothesis that the effective charge distribution is locked to the circulating phase pattern is an additional structural assumption.

It is represented by

$$\rho(\varphi, t) = \rho(\varphi - \omega t)$$

From this assumed rotating distribution follows the effective current density

$$\mathbf{J}_Q = \rho \mathbf{v}.$$

and, for tangential motion,

$$v_\varphi = \omega r$$

The crucial point is that the existence of the charge e is experimental, but its phase-locked circulation is part of the supertorus hypothesis.

12.9 Recovery of the Bohr Magnetron

If the effective charge circulation is characterized by radius R , the magnetic moment is

$$\mu = \frac{e\omega R^2}{2}$$

Using

$$\omega R = c$$

and

$$R = \frac{\hbar}{m_e c},$$

this gives

$$\mu = \frac{e\hbar}{2m_e} = \mu_B$$

This is an important consistency result.

However, it is again not completely independent, because R was obtained using $\hbar$ through the boundary condition $U = \hbar\omega$.

What is nontrivial is that the same value of R , obtained from the energy and phase conditions, also produces the correct leading magnetic-moment scale when combined with the charge e .

12.10 What Is Assumed About the Charge Asymmetry

The measured electron magnetic moment requires

$$\langle r^2 \rangle_Q = (1 + a_e)R^2$$

This is a constraint derived from the measured magnetic moment.

In the simplified two-region representation, it implies

$$q_{\text{in}} \approx 0.5613 e, \quad q_{\text{out}} \approx 0.4387 e$$

These fractions are therefore **not predicted independently**; they are inferred from μ_e within the two-region approximation.

By contrast, the relation

$$E(r) \propto r^{-1/2}$$

was introduced as a geometrical scaling hypothesis.

Combining that hypothesis with

$$q(r) \propto E(r)$$

gave

$$q_{\text{in}} \approx 0.5635 e, \quad q_{\text{out}} \approx 0.4365 e$$

The closeness of these numbers to the values required by μ_e is therefore a genuine numerical comparison within the simplified model, although it rests on the unproven scaling assumptions.

12.11 The Non-Radiation Condition

Another central assumption is that the closed phase structure suppresses net outward radiation.

The required physical condition is

$$\lim_{r \rightarrow \infty} \oint_{A(r)} \langle \mathbf{S} \rangle \cdot d\mathbf{A} = 0.$$

The cancellation of equally weighted phase components distributed over 2π provides a plausible mechanism for suppressing the leading far-field radiation.

However, exact non-radiation of the complete three-dimensional supertorus has **not yet been derived**.

It remains one of the principal conditions that a full Maxwell solution must satisfy.

12.12 Summary of Logical Status

The logical structure of the minimal model may therefore be summarized as follows.

Experimental inputs:

$$m_e, \quad e, \quad J = \frac{\hbar}{2}, \quad \mu_e$$

together with the energy-frequency relation

$$U = \hbar\omega.$$

Structural assumptions:

closed toroidal-cell geometry

$$\Delta\phi_{\text{total}} = 2\pi$$

$$\omega = ck$$

$$U_{\text{in}} = U_{\text{out}} = \frac{U}{2}$$

$$\rho(\varphi, t) = \rho(\varphi - \omega t)$$

and, for the first charge-asymmetry estimate,

$$E(r) \propto r^{-1/2}, \quad q(r) \propto E(r).$$

Derived quantities within these assumptions:

$$R = \frac{\hbar}{m_e c}$$

$$a = \frac{R}{4}$$

$$V = 2\pi^2 R a^2$$

$$E_0 \sim 5.1 \times 10^{17} \text{ V m}^{-1}$$

$$B_0 \sim 1.7 \times 10^9 \text{ T}$$

and the leading magnetic-moment scale

$$\mu \sim \mu_B.$$

12.13 What Would Count as a Genuine Prediction

The strongest future version of the model would require fewer experimentally inserted quantities.

For example, if a complete electromagnetic solution determined a/R without using the measured electron spin, then the resulting angular momentum could be compared independently with

$$\frac{\hbar}{2}.$$

Likewise, if the field solution determined $\rho(r)$ without using μ_e , then the magnetic moment obtained from

$$\mu = \frac{1}{2} \int_V \mathbf{r} \times \mathbf{J}_Q dV$$

would become a genuine prediction.

Similarly, exact cancellation of the radiative field derived directly from Maxwell's equations would convert the present non-radiation assumption into a calculated property.

The scientific value of the present minimal model therefore lies not in claiming that all electron properties have already been derived from electromagnetism, but in identifying a **specific and highly constrained field configuration** for which such a derivation can now be tested.

12.14 Central Methodological Principle

The model should therefore be judged according to a simple rule:

An input may constrain the model, but it cannot also count as its prediction.

This principle prevents numerical agreement from being overstated.

At the same time, when several independent inputs constrain one common geometry and that geometry subsequently produces additional correct relationships without further adjustment, those relationships provide meaningful tests of internal consistency.

The next section, **13. Open Problems and Falsifiability**, can therefore focus precisely on the calculations that remain capable of confirming or rejecting the supertorus hypothesis rather than merely refining its parameters

13. Open Problems and Falsifiability

The preceding sections show that the minimal supertorus model can be constrained by several known properties of the electron. However, internal numerical consistency is not sufficient to establish the model as a physical description of the electron.

A useful physical model must also expose itself to failure.

The purpose of this section is therefore to identify the principal unresolved problems and, wherever possible, formulate them as quantitative tests. A complete supertorus theory should not merely reproduce quantities already used as boundary conditions. It should generate additional results that can be compared with experiment or with exact electromagnetic calculations.

The most important unresolved questions concern non-radiation, the existence of a self-consistent field solution, the origin and distribution of charge, angular momentum, magnetic moment, stability, and the relation between the internal phase motion and physical energy transport.

13.1 Exact Non-Radiation

The first and most fundamental requirement is that the proposed closed electromagnetic structure must not continuously lose energy through radiation.

For a stationary electron, the total time-averaged outward electromagnetic power must vanish,

$$P_{\text{rad}} = \lim_{r \rightarrow \infty} \oint_{A(r)} \langle \mathbf{S} \rangle \cdot d\mathbf{A} = 0.$$

The phase-closure argument developed earlier suggests that radiation from different parts of the supertorus may cancel by superposition. For a continuously distributed phase,

$$\int_0^{2\pi} e^{i\phi} d\phi = 0.$$

This demonstrates cancellation of an idealized set of equal-amplitude phase components. It does not, by itself, prove that all radiative multipole components of the complete three-dimensional electromagnetic structure vanish.

The stronger requirement is that the far-field terms themselves cancel,

$$\mathbf{E}_{\text{rad}}(\mathbf{r}, t) \rightarrow 0, \quad \mathbf{B}_{\text{rad}}(\mathbf{r}, t) \rightarrow 0 \quad (r \rightarrow \infty).$$

This provides a clear falsification criterion: **if the proposed phase-locked current distribution necessarily produces a nonzero time-averaged radiative flux, the stationary supertorus cannot represent a stable electron.**

13.2 Complete Maxwell Field Solution

The present analysis uses simplified descriptions of the internal field distribution. Ultimately, the electric and magnetic fields must satisfy Maxwell's equations throughout the relevant regions.

In differential form,

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

and

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

The full solution must also satisfy the appropriate boundary and continuity conditions.

The central mathematical problem can therefore be formulated as follows:

Does there exist a finite, closed, phase-locked electromagnetic field configuration with the proposed toroidal topology that simultaneously satisfies Maxwell's equations, the electron boundary conditions, finite total energy, and zero net radiation?

13.3 Self-Consistency of the Charge Distribution

At present, the electron charge is introduced as a boundary condition,

$$\int_V \rho \, dV = -e$$

But the detailed distribution $\rho(r,t)$ has not been derived.

A complete solution should determine ρ consistently with the electric field through Gauss's law,

$$\rho = \epsilon_0 \nabla \cdot \mathbf{E}.$$

This is a much stronger condition than assigning charge fractions to the inner and outer regions.

Once $E(r,t)$ is known, the model should determine

$$Q_{\text{calc}} = \epsilon_0 \int_V \nabla \cdot \mathbf{E} \, dV.$$

The boundary condition requires

$$Q_{\text{calc}} = -e$$

If no self-consistent field configuration can simultaneously produce the required charge and the assumed closed internal wave structure, the model would have to be revised.

13.4 Origin of the Inner–Outer Asymmetry

The preceding section found that the magnetic moment requires an effective radial charge asymmetry.

In the two-region approximation,

$$q_{\text{in}} \approx 0.5613 \, e, \quad q_{\text{out}} \approx 0.4387 \, e$$

A simple geometrical scaling produced similar values, but that scaling was not derived from the complete electromagnetic equations.

A stronger theory must obtain the radial dependence directly:

$$\rho = \rho(r, \theta, \varphi, t)$$

The calculated distribution can then be tested against the magnetic-moment constraint,

$$\frac{1}{e} \int_V |\rho| r^2 \, dV = (1 + a_e) R^2$$

This would convert the present approximate agreement into a genuine quantitative test.

13.5 Angular Momentum Without Fitting the Minor Radius

The present value

$$a = \frac{R}{4}$$

was obtained by imposing the known electron angular momentum,

$$J = \frac{\hbar}{2}$$

Consequently, the present calculation does not independently predict electron spin.

A stronger test would reverse the calculation.

If a complete electromagnetic solution independently determines a , its angular momentum should then be calculated from the Poynting vector,

$$\mathbf{J}_{\text{EM}} = \frac{1}{c^2} \int_V \mathbf{r} \times \mathbf{S} \, dV.$$

The model would then predict a value

$$J_{\text{calc}} = |\mathbf{J}_{\text{EM}}|.$$

Only if

$$J_{\text{calc}} = \frac{\hbar}{2}$$

emerges without using electron spin to determine the geometry would the spin magnitude become an independent prediction of the model.

13.6 Magnetic Moment Without Using μ_e

The same principle applies to the magnetic moment.

At present, the measured magnetic moment constrains the radial charge distribution. A complete model should instead calculate the charge-current density,

$$\mathbf{J}_Q(\mathbf{r}, t),$$

and then determine

$$\mu_{\text{calc}} = \frac{1}{2} \int_V \mathbf{r} \times \mathbf{J}_Q dV.$$

The calculated value could then be compared with

$$|\mu_e| = (1 + a_e)\mu_B$$

This would be substantially more demanding than recovering the Bohr-magneton scale from the already fixed radius R .

In particular, reproducing the small anomaly

$$a_e \approx 1.16 \times 10^{-3}$$

without inserting it as an input would provide a stringent test of the detailed field structure.

13.7 Phase Velocity Versus Energy Transport

The synchronous phase construction produced

$$v(r) = \omega r$$

With

$$a = \frac{R}{4},$$

the outer phase velocity becomes

$$v_{\text{out}} = \frac{5}{4}c$$

This is acceptable only if v_{out} describes phase progression rather than superluminal transport of energy, charge, or information.

The actual electromagnetic energy-flow velocity can be characterized locally by the ratio of the Poynting flux to the energy density,

$$\mathbf{v}_E = \frac{\mathbf{S}}{u_{\text{EM}}}.$$

where

$$u_{\text{EM}} = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}.$$

A complete solution should therefore distinguish explicitly between

$$v_{\text{phase}}, \quad v_E, \quad v_Q,$$

where v_Q represents any physically meaningful charge-transport velocity.

The model would face a serious difficulty if its magnetic moment required literal transport of localized charge faster than c , rather than only a superluminal phase pattern.

13.8 Stability

A non-radiating solution is not automatically a stable solution.

A physical electron model must remain localized under small perturbations. Let the equilibrium field configuration be denoted schematically by

$$\mathbf{E}_0(\mathbf{r}, t), \quad \mathbf{B}_0(\mathbf{r}, t).$$

Introduce small perturbations,

$$\mathbf{E} = \mathbf{E}_0 + \delta\mathbf{E}, \quad \mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}.$$

A stable mode requires the perturbations not to grow without bound,

$$|\delta\mathbf{E}(t)|, |\delta\mathbf{B}(t)| \not\rightarrow \infty.$$

This stability analysis may prove more restrictive than the static energy calculation.

If the closed field structure immediately expands, collapses, or radiates after an arbitrarily small perturbation, it cannot represent a stable free electron.

13.9 Mechanical Stresses Are Not Sufficient

The model should not introduce an unspecified mechanical material or external confining force merely to prevent the electromagnetic structure from expanding.

The electromagnetic stress is described by the Maxwell stress tensor,

$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right).$$

A self-contained electromagnetic model should demonstrate that the integrated stresses are compatible with the assumed stationary geometry.

Schematically, equilibrium requires

$$\mathbf{F}_{\text{net}} = \oint_A \mathbf{T} \cdot d\mathbf{A} = 0.$$

If additional non-electromagnetic stresses are required, they must be identified explicitly rather than hidden inside the geometry.

13.10 External Coulomb Field

A particularly important unresolved problem is the coexistence of a closed internal electromagnetic wave with the external electric field of the electron.

Far from the electron, the electric field must approach the Coulomb form,

$$\mathbf{E}(r) \rightarrow -\frac{e}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}}.$$

At the same time, the oscillating internal field must not produce a radiative $1/r$ component.

Thus, asymptotically, the model requires

$$\mathbf{E} = \mathbf{E}_{\text{Coulomb}} + \mathbf{E}_{\text{rad}}, \quad \mathbf{E}_{\text{rad}} \rightarrow 0.$$

This distinction is fundamental: **the electron may possess a nonzero static external field while still having zero net outgoing radiation.**

A complete solution must demonstrate both simultaneously.

13.11 External Magnetic Dipole Field

Likewise, the circulating charge must generate the observed magnetic dipole field.

At sufficiently large distance, the field should have the dipole form

$$\mathbf{B}(\mathbf{r}) \rightarrow \frac{\mu_0}{4\pi r^3} [3(\boldsymbol{\mu} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \boldsymbol{\mu}].$$

with

$$|\boldsymbol{\mu}| = |\mu_e|$$

The model therefore has to generate three qualitatively different field components:

$$E_{\text{Coulomb}} \propto \frac{1}{r^2},$$

$$B_{\text{dipole}} \propto \frac{1}{r^3},$$

while suppressing the radiative component

$$E_{\text{rad}}, B_{\text{rad}} \propto \frac{1}{r}.$$

This provides a particularly clear mathematical target for a future field solution.

13.12 The Strongest Falsification Test

The central test can now be formulated compactly.

Seek a field configuration

$$\{\mathbf{E}, \mathbf{B}, \rho, \mathbf{J}\}$$

with the supertorus topology and the phase closure developed in this paper.

Without independently fitting each electron property, that configuration should simultaneously satisfy

$$U_{\text{calc}} = m_e c^2,$$

$$Q_{\text{calc}} = -e,$$

$$J_{\text{calc}} = \frac{\hbar}{2},$$

$$|\mu_{\text{calc}}| = |\mu_e|,$$

and

$$P_{\text{rad}} = 0.$$

In addition, the solution must be dynamically stable.

This is a highly restrictive set of conditions. Failure of any indispensable condition would falsify at least the corresponding version of the supertorus model.

13.13 A Hierarchy of Tests

The future development of the model can therefore proceed in increasing levels of difficulty.

The **first test** is geometrical non-radiation by superposition. The question is whether the complete 2π phase distribution cancels all far-field radiation rather than only its simplest component.

The **second test** is a continuous electromagnetic calculation across the toroidal cross-section.

The **third test** is recovery of the correct external Coulomb and magnetic dipole fields.

The **fourth test** is calculation of angular momentum and magnetic moment from those fields without using J and μ_e to adjust the geometry.

The **fifth and strongest test** is dynamical stability of the resulting solution.

Each stage can reject the model independently.

13.14 What Would Constitute Strong Evidence

Conversely, the evidential status of the model would change substantially if a single self-consistent solution were found that satisfied several conditions not imposed in advance.

For example, suppose the field equations and topology determined the ratio

$$\frac{a}{R}$$

without using electron spin.

If the subsequent field integral yielded

$$J_{\text{calc}} \approx \frac{\hbar}{2},$$

that would constitute an independent result.

Likewise, if the same solution produced a charge distribution from which

$$|\mu_{\text{calc}}| \approx 1.00116 \mu_B$$

followed without inserting the electron magnetic moment, the agreement would be considerably more significant than the present boundary-condition calculation.

Most importantly, these results would have to arise from the **same field configuration**, not from separate approximations adjusted for each property.

13.15 Status of the Supertorus Hypothesis

The calculations developed in this paper therefore place the supertorus hypothesis in an intermediate position.

It is more than a purely qualitative geometrical picture because definite scales and integral constraints have been obtained:

$$R = \frac{\hbar}{m_e c}, \quad a \approx \frac{R}{4},$$

$$E_0 \sim 10^{17} \text{ V m}^{-1}, \quad B_0 \sim 10^9 \text{ T},$$

and the geometry naturally produces a magnetic-moment scale of order

$$\mu \sim \mu_B.$$

At the same time, it is not yet a complete electromagnetic theory of the electron. The detailed fields, charge distribution, exact non-radiation, and stability have not been derived from a single self-consistent solution.

The appropriate conclusion is therefore not that the electron has been derived from the supertorus, but that the hypothesis has become sufficiently constrained to be **calculable and falsifiable**.

That is an important transition. A geometrical idea becomes a physical model only when it specifies calculations that can fail.

The decisive next step is consequently clear:

construct the field →
calculate its properties →
compare with the electron.

This provides a natural endpoint for the quantitative development of the present paper and a starting point for a more complete electromagnetic treatment of the supertorus.

14. Discussion

The analysis developed in the preceding sections suggests that the supertorus may provide more than a purely geometrical representation of the electron. Starting from a compact toroidal structure and a limited set of physical assumptions, several characteristic properties of the electron can be brought into a common geometrical and electrodynamic framework.

The central result is not any single equation, but the fact that quantities that are usually introduced independently begin to appear as parts of the same structure. The electron charge, angular momentum, magnetic moment, characteristic dimensions, internal electric and magnetic fields, circulating charge,

and the asymmetry between the inner and outer regions of the torus can all be discussed within one model.

This is particularly significant because the construction does not begin by assigning a separate mechanism to each observable property. Instead, the same circulating electromagnetic configuration participates in several of them simultaneously. Geometry determines the possible paths of motion; charge circulation produces magnetic effects; electromagnetic fields contribute to the internal dynamics; and the dimensions of the structure enter directly into the conditions required for angular momentum and magnetic moment.

The resulting picture therefore points toward a possible unification of properties that are normally treated as intrinsic parameters of a point-like electron.

14.1 From parameters to structure

In the standard description of the electron, quantities such as charge, mass, spin, and magnetic moment are experimentally well-defined characteristics. The supertorus approach asks a different question: could at least some of these quantities arise from an underlying electromagnetic geometry?

The calculations presented here indicate that this question can be formulated quantitatively.

The minor and major radii are not merely geometrical decorations. They enter into the expressions for angular momentum, field amplitudes, circulating charge, and magnetic moment. Consequently, the geometry is constrained by the physical properties that the model attempts to reproduce.

For example, angular momentum connects a characteristic radius and momentum through

$$L = rp.$$

For electromagnetic energy propagating with characteristic speed c ,

$$p = \frac{E}{c},$$

and therefore

$$L = \frac{rE}{c}.$$

The electron spin condition requires

$$L = \frac{\hbar}{2}.$$

Thus, the characteristic dimensions and the circulating energy cannot be chosen independently.

This represents an important transition. A geometrical model becomes physically interesting when its dimensions and internal dynamics are constrained by measurable quantities. The supertorus construction begins to satisfy this requirement.

14.2 A network of mutually connected constraints

One of the most encouraging features of the model is that its equations do not remain isolated.

A change in the assumed geometry affects the charge distribution. The charge distribution affects the electric and magnetic fields. The circulating charge determines the magnetic moment. The distribution of energy and momentum constrains the angular momentum. The inner-outer asymmetry modifies the local field configuration.

The electron properties therefore form a network of coupled constraints rather than a collection of unrelated fitting conditions.

Schematically,

Geometry → charge distribution →
fields →
momentum and magnetic moment.

At the same time, the experimentally known electron properties constrain the same chain in the opposite direction:

measured electron properties →
allowed fields and currents →
allowed geometry.

The supertorus must satisfy both directions simultaneously.

This mutual restriction is potentially one of the strongest features of the approach. The more independently known electron properties that can be reproduced by the same small set of geometrical parameters, the less arbitrary the model becomes.

14.3 The role of the two characteristic radii

The existence of two geometrically distinct radii is especially important.

Let the major radius be R and the minor radius a .

The corresponding characteristic path lengths are

$$\ell_R = 2\pi R$$

and

$$\ell_a = 2\pi a,$$

If the characteristic propagation speed along these paths is c , the circulation times are

$$f_R = \frac{c}{2\pi R}$$

and

$$T_a = \frac{2\pi a}{c},$$

The corresponding characteristic frequencies are

$$f_R = \frac{c}{2\pi R}$$

and

$$f_a = \frac{c}{2\pi a}.$$

The presence of two characteristic length scales and two circulation frequencies provides a natural way of accommodating more than one internal dynamical scale within a single geometrical object.

Rather than assigning every electron property to a separate internal mechanism, the supertorus allows different observables to emerge from different aspects of the same double-circulation geometry.

14.4 Electromagnetic self-organization

The calculations also suggest a broader physical interpretation.

A circulating charge produces a current,

$$I = \frac{q}{T}.$$

For circulation around a path of radius R ,

$$T = \frac{2\pi R}{c},$$

so that

$$I = \frac{qc}{2\pi R}.$$

A circulating current produces a magnetic moment,

$$\mu = IA,$$

where

$$A = \pi R^2.$$

Consequently,

$$\mu = \frac{qcR}{2}.$$

This simple relation illustrates the central idea: geometry, charge circulation, and magnetic moment are not necessarily separate ingredients but different aspects of the same physical construction.

Time-dependent electric fields introduce displacement current,

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt},$$

and the magnetic field follows the Ampère–Maxwell relation,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(I + \varepsilon_0 \frac{d\Phi_E}{dt} \right).$$

The electromagnetic energy density is

$$u = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2,$$

and the total field energy is

$$U = \int_V u dV.$$

The electromagnetic configuration therefore carries energy, momentum, and angular momentum.

This suggests that the fields need not merely surround an already existing particle. They may participate in forming and maintaining the particle-like structure itself.

Instead of only

Electron $\rightarrow$ electromagnetic field,

the supertorus suggests the complementary possibility

self-organized electromagnetic structure $\rightarrow$ electron properties.

14.5 Why the asymmetry matters

The inner–outer asymmetry of a toroidal structure deserves particular attention.

For a point in the minor cross-section described by angle θ , the distance from the symmetry axis is

$$\rho(\theta) = R + a \cos \theta,$$

The outer side corresponds to

$$\rho_{\text{out}} = R + a,$$

whereas the inner side corresponds to

$$\rho_{\text{in}} = R - a.$$

Therefore,

$$\rho_{\text{out}} \neq \rho_{\text{in}}.$$

A torus is consequently not locally equivalent on its inner and outer sides. The path length, curvature, field distribution, and charge density need not be identical across the minor cross-section.

The asymmetry discussed in Section 10 may therefore be a necessary part of the structure rather than merely a small correction to an otherwise symmetric model.

Some electron properties may depend not only on the total circulating charge or total field energy but also on how these quantities are distributed within the toroidal geometry.

A more complete treatment of this asymmetry could therefore become an important next stage of the model.

14.6 Toward a geometrical description of the electron

The main achievement of the present analysis is that it identifies a coherent direction in which the electron can be investigated as a finite electromagnetic structure.

A complete description would require a self-consistent connection between the charge density

$$\rho(\mathbf{r}, t),$$

the current density

$$\mathbf{J}(\mathbf{r}, t),$$

and the electromagnetic fields

$$\mathbf{E}(\mathbf{r}, t), \quad \mathbf{B}(\mathbf{r}, t).$$

These quantities would have to satisfy Maxwell's equations together with the experimentally known global properties of the electron.

The objective of the model can therefore be expressed schematically as

$$\{R, r, \rho, \mathbf{J}, \mathbf{E}, \mathbf{B}\} \longrightarrow \left\{e, m_e, \frac{\hbar}{2}, \mu_e\right\}.$$

In other words, the geometrical and electromagnetic quantities on the left should collectively account for the experimentally known electron properties on the right.

The calculations presented in this work suggest that such a program is sufficiently structured to be pursued quantitatively.

The model therefore provides more than an illustration. It defines a research program.

The next step is not to introduce separate mechanisms for every remaining electron property. The more demanding and scientifically more interesting objective is to reduce the number of independent assumptions and determine whether an increasingly self-consistent solution emerges from geometry, Maxwell electrodynamics, and the experimentally known constants of the electron.

If this can be achieved, quantities currently introduced as independent intrinsic properties may acquire a common geometrical and electromagnetic origin.

That possibility is the principal motivation for continuing the investigation of the supertorus.

15. Conclusions

The purpose of this work was to examine whether a finite toroidal electromagnetic structure can reproduce, or at least connect, several fundamental properties of the electron within a single geometrical framework.

The analysis shows that the supertorus is capable of establishing such connections. Starting from a geometry characterized primarily by the major radius R and the minor radius r , the model links charge circulation, electromagnetic fields, angular momentum, magnetic moment, internal energy, and charge asymmetry.

The significance of the result lies not in reproducing one isolated electron parameter, but in the emergence of a **system of mutually connected constraints**.

15.1 A common geometrical framework

The electron is experimentally characterized by several precisely known quantities:

$$e, \quad m_e, \quad \frac{\hbar}{2}, \quad \mu_e.$$

In a point-particle description, these quantities enter as intrinsic properties of the electron.

The supertorus approach explores the possibility that at least some of them may instead be manifestations of an underlying geometry and electromagnetic dynamics.

The basic idea can be summarized as

Geometry + electromagnetic circulation → observable electron properties.

The analysis performed in this work shows that this idea can be developed quantitatively.

The characteristic dimensions of the supertorus enter directly into the conditions for angular momentum and magnetic moment. At the same time, the electromagnetic fields determine energy and momentum, while the circulating charge generates current and magnetic effects.

These are therefore not independent parts of the model. They belong to the same physical structure.

15.2 Angular momentum and magnetic moment

Two particularly important constraints are provided by the angular momentum and magnetic moment of the electron.

The angular momentum condition is

$$L = \frac{\hbar}{2}.$$

For circulating electromagnetic energy,

$$p = \frac{E}{c},$$

so that a characteristic contribution to angular momentum has the form

$$L = rp = \frac{rE}{c}.$$

The electron spin therefore imposes a direct relation between energy and geometry:

$$\frac{rE}{c} = \frac{\hbar}{2}.$$

Similarly, a circulating charge produces a current

$$I = \frac{q}{T}.$$

For circulation with characteristic radius R ,

$$T = \frac{2\pi R}{c},$$

and therefore

$$I = \frac{qc}{2\pi R}.$$

With

$$A = \pi R^2,$$

the corresponding magnetic moment is

$$\mu = IA = \frac{qcR}{2}.$$

These relations demonstrate an important feature of the supertorus approach: angular momentum and magnetic moment are no longer entirely separate properties. Both are connected to circulation and to the geometry of the same structure.

The electron values

$$L = \frac{\hbar}{2}$$

and

$$\mu_e$$

therefore become simultaneous constraints on the allowed supertorus configuration.

15.3 The electron as an electromagnetic structure

The calculations developed in this work support a physical picture in which the electromagnetic field is not merely an external field produced by an otherwise structureless electron.

Instead, the field itself may participate in the electron structure.

Its energy density is

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2,$$

and its total energy is

$$U = \int_V u dV.$$

If the electron rest energy is ultimately to be represented electromagnetically, the model must approach the condition

$$U = m_e c^2.$$

Likewise, the electromagnetic momentum and its circulation must reproduce the required angular momentum,

$$L = \frac{\hbar}{2}.$$

The circulating charge and current must simultaneously reproduce the magnetic moment,

$$\mu = \mu_e.$$

Thus the desired self-consistent solution must satisfy, at minimum,

$$U = m_e c^2,$$

$$L = \frac{\hbar}{2},$$

$$q = -e,$$

and

$$\mu = \mu_e.$$

The strength of the model will therefore depend increasingly on whether all these conditions can be satisfied **simultaneously by the same electromagnetic configuration**.

15.4 The importance of the supertorus geometry

The supertorus introduces two characteristic radii,

$$R \quad \text{and} \quad r,$$

and therefore two geometrically different circulations. This distinction may be essential.

The major circulation provides one characteristic scale,

$$\ell_R = 2\pi R,$$

while the minor circulation provides another,

$$\ell_r = 2\pi r.$$

Their corresponding characteristic frequencies are

$$f_R = \frac{c}{2\pi R}$$

and

$$f_r = \frac{c}{2\pi r}.$$

The resulting double-circulation geometry provides substantially more internal structure than a simple ring or spherical shell while retaining a small number of fundamental geometrical parameters.

Moreover, the toroidal geometry naturally contains an inner–outer asymmetry,

$$R - r \neq R + r,$$

which may influence the charge density, field strength, energy distribution, and local momentum density.

This asymmetry may ultimately prove important for obtaining a fully self-consistent electron configuration.

15.5 From measured constants toward derived properties

Perhaps the most interesting direction opened by the supertorus model concerns the status of the electron constants themselves.

At present, the electron is characterized experimentally by quantities such as

$$e, \quad m_e, \quad \hbar, \quad \mu_e.$$

A deeper structural model should attempt to explain why these quantities occur together with precisely their observed values.

The long-term objective can therefore be written schematically as

$$\begin{array}{c} \{R, r, \rho(\mathbf{r}, t), \mathbf{J}(\mathbf{r}, t), \mathbf{E}(\mathbf{r}, t), \mathbf{B}(\mathbf{r}, t)\} \\ \Downarrow \\ \left\{ e, m_e, \frac{\hbar}{2}, \mu_e \right\}. \end{array}$$

The transition represented by this arrow is the central challenge for further development.

The ultimate goal would be to reduce the number of independently inserted electron properties and replace them, wherever possible, by quantities derived from the

internal geometry and electromagnetic dynamics.

In that sense, the supertorus model proposes a transition from

measured constants as input

toward

measured constants as consequences of structure.

15.6 A direction for further development

The present work does not yet constitute a complete dynamical theory of the electron. Several elements still require a more rigorous treatment, particularly the three-dimensional charge and current distributions, the self-consistent Maxwell field solution, the stability of the configuration, and the detailed relationship between the two circulations.

These remaining tasks, however, now have a more definite mathematical context.

The problem is no longer simply to imagine a possible shape of the electron. It is to determine whether a specific electromagnetic geometry can satisfy a tightly connected set of experimentally imposed constraints.

This distinction is important.

The supertorus presented here is therefore best regarded not merely as a geometrical picture, but as the beginning of a quantitative structural model.

Its development should proceed by progressively replacing assumptions with derived relations and by requiring every refinement to remain consistent with the experimentally known properties of the electron.

The calculations presented in this paper provide reasons to pursue that question.

They show that geometry, electromagnetic energy, charge circulation, angular momentum, and magnetic moment can be placed within a single interconnected framework. They also identify the quantities

that a more complete calculation must determine rather than assume.

The supertorus thus offers a concrete direction for exploring whether the familiar properties of the electron may have a common structural origin.

If such a connection can be completed, the electron would no longer need to be viewed

only as a point carrying a collection of independent intrinsic properties. Its measured characteristics could instead be understood as different observable manifestations of one underlying electromagnetic geometry.

This is the direction suggested by the present analysis—and the principal result of this work.